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Mathematical Modeling of Nigeria's Zero-Coupon Yield Curve



CENTRAL BANK OF NIGERIA

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APPRECIATION*

To the glory of God, the author expresses profound gratefulness to the entire management of the Central Bank of Nigeria, for the privilege given to serve, and for the opportunity extended to, somehow, give something back to Nigeria for giving me the federal scholarship to study abroad and to realize my potentials. With this token of contribution, let it be said that there can be no greater honor for a man than to be called upon to serve his country. It is not about what can be acquired; it is about what can be selflessly given.

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INTRODUCTION

1.1 Project Overview

Over the past ten years, the Nigerian economy, particularly its finance sector, has been fundamentally transformed and strengthened. At present, Nigerian financial institutions are fast becoming major players in the regional and global financial arena. The emergence of a vibrant finance sector within the Nigerian economy has engendered a more meaningful and diversified way of sourcing capital to finance her economic growth and development. The finance sector has been sustained mainly through the consolidation of the capitalization base of Nigerian financial institutions. Regrettably, however, the bulk of development capital has almost been entirely and dangerously limited to equity financing. This is flawed in several ways.

Firstly, equity financing represents a fraction of investment capital, which the Nigerian economy could garner, judging from the aggregate monetary supply within her economy, and the magnitude of Foreign Indirect Investment (including remittances available from the diaspora) from outside the country. Focusing mainly on equity financing significantly limits the availability of capital available and inhibits economic growth. Secondly, over-reliance on equity financing means that other meaningful and modern sources of corporate financing are being ignored and untapped, thereby limiting the diversity and strength of Nigeria's financial sector and, hence, its ability to compete in the global economic arena. Thirdly, over-reliance on equity financing denies Nigerian capital market investors the benefits of portfolio diversification and the opportunity of meaningful horizontal risk spreading and dispersal. This makes the entire monetary system vulnerable and dangerously exposed to systemic risk. This, in turn, adds to the cost of financing from abroad, as it impacts negatively on Nigeria's rating outcome. Fourthly, by relying on equity financing alone, Nigeria considerably narrows and limits variation in the number of financial transactions her institutions can engage in, and, consequently, the multi-billion naira transaction fees likely to be accruable to the Nigerian Stock Exchange. This designates Nigeria's economy as not-investable by major institutional and portfolio investors from across the globe.

Equity financing lacks all the desirable attributes of modern financing tools. These include ability to choose between over-the-counter (OTC) and exchange floor transactions; the possibility to convert or attach equity financial assets to warrants; the opportunity to pool, tranche and sell-on debt instruments using, asset-backed securitization techniques, and many more possibilities. Where such an equity market focuses only on plain vanilla products, these limitations become more acute and frightening to investors.

Nigeria's over-reliance on equity financing also limits the degree, as well as the level, of sophistication of Nigeria's financial system and that of her finance practitioners. Limited experience and exposure to modern investment techniques will, undoubtedly, not stand Nigeria in a good stead when it comes to engineering and structuring new financial instruments to take up rare opportunities, both locally and abroad. Nigeria is uniquely positioned to gain far more from trading financial products and instruments than from trading tangible oil and cassava, for example.

The depth of development capital currently available within the Nigerian economy is extremely shallow and insufficient to sustain her socio-economic requirements. As such, the capital base of the Nigerian economy needs to be strengthened, diversified and integrated into the global financial markets.

The most feasible and sustainable alternative source of financing applicable to the Nigerian economy at this stage of her growth cycle is debt financing. Complementing the Nigerian equity market with an efficient and well regulated bond market will further enhance and substantially deepen Nigeria's capital market, and usher-in the much needed alternative sources of capital to accelerate the pace of her economic and financial growth and development.

1.2 Research Objectives of the Project

Arising from the foregoing remarks, the main objectives of this research project are to prepare the foundation needed for Nigeria to diversify into debt-financing. Specifically, this research endeavour seeks to calibrate and fit Nigeria's Zero-Coupon Bond Yield Curve. The work also seeks to explore various ways in which the Zero-Coupon Yield Curve could be beneficial to the banking and financial system of the Nigerian economy. Particular emphasis will be paid to applications of yield curves in monetary policy enhancement. To this end, the main objectives of this research work are as follows:

- To calibrate the yield value of Nigeria's Zero-Coupon bonds,
- To fit Nigeria's Zero-Coupon yield curve,
- To critically analyze and interpret Nigeria's Zero-Coupon yield curve, and
- To espouse the virtues of applying the Zero-Coupon yield curve in Nigeria, and highlight the pre-requisites for realizing this objective.

1.3 Research Methodology

In delivering the above-referenced objectives, extensive analysis of appropriate research inputs from secondary sources (including academic journals and related published materials) shall be perused and critically explored. The aim here is to enrich the research work with up-to-date advancements in yield-curve modeling.

Carefully structured discussion and brainstorming sessions with finance experts and market practitioners in the areas of the yield curve and interest rate modeling will also be employed. This is to ensure that theoretical, as well as practical issues are comprehensively and exhaustively considered when making inferences, and also at the recommendation stage of the work. This will be complemented with an extensive use of financial information terminals, including those of Bloomberg and Reuters. The essence of this is to facilitate the simulation, as well as other technical analyses of data input.

In addition, empirical data and evidence about the yield curves of other countries, particularly those of the United Kingdom, France, Spain, India, Greece, Ireland and Germany shall be critically analyzed and benchmarked against the yield curve of Nigeria. The essence of this is to verify macro-economic indicators (including interest rates, level of inflation as well as borrowing cost differentials) obtained from other sources against the yield curve interpretation exhibited by Nigeria's Zero-Coupon yield curve.

1.4 Literature Review

The yield curve has been extensively studied by experts in both academic and empirical realms. The bulk of the investigations centres mainly on two strands – analysis of the shape of the yield curve, and the application of the yield curve in financial policy enhancement. Other groups of researchers investigate the usefulness of the yield curve in predicting economic crises and the long-term wellbeing of a nation's financial system.

Prominent among such researchers is Wu, (2001), whose work explores the effect of monetary policy activities on the slope factor of the yield curve. Using empirical evidence from the Federal Reserve Bank of San Francisco, Wu contends that central bank monetary actions exert enormous, but short-lived, influence on the slope of the yield curve. Wu contends that about 80% of movements in the slope of a yield curve are attributable to central banks' monetary policy interventions. Wu's work is consistent with the findings of Ilmanen and Iwanowski, (1997).

Evans and Marshall (2001) examine the same problem, using a different approach. They developed several models with rich macroeconomic dynamics and analyzed how the "level", "slope" and "curvature" factors are affected by the structural shocks identified in their models. Their work corroborates the work of Ang and Piazzesi (2001) who argue that a substantial portion of short and medium-term bond yields is influenced by the dynamics of national macroeconomic variables.

The work of Evans and Marshall (2001) takes the research further. The authors affirm that, more than anything else, macroeconomic variables do influence much of the movements in long-term bond yields, particularly in the long run.

The two authors' empirical finding confirms that changes in household consumption patterns and preferences do induce substantial and persistent shifts in the level and slope of the yield curve. The findings of Evans and Marshall are largely supported by Lardic, et.al. (2003).

Ang and Piazzesi (2001) investigate other features of the yield curve. Their work investigates applications of the yield curve in understanding and managing the effects of inflation and real economic activity (such as advancements in technology) on the yield curve in an asset pricing framework. In their model, the two authors effortlessly demonstrate that bond yields are determined not only by level, slope and curvature of the yield curve, but also by a measure of inflation and real economic activities. The authors postulate that while short-term and medium-term movements in the yield (usually up to one year) are attributable to inflationary and real economic activities, most of the long-term movements in the yield curve are as a result of the influence of level, slope and curvature of the curve. The authors conclude that incorporating inflation and real economic activities into

their model significantly improves their ability to forecast the yield curve's movement.

Douglas Elliot (2004) takes possible applications of the yield curve to another level. His work investigates application of the yield curve in managing the liability-driven investment attitude of pension funds. Using empirical data from the American Pension Benefit Guaranty Corporation, this author maintains that holding investment portfolios whose future returns replicate future liabilities provides a very convenient and effective approach towards managing such liabilities. As the yield curve provides a barometer through which future returns on fixed-income investments can be measured, this author posits that it serves a very useful purpose to align such future returns with future liabilities of the investors. This would help to minimize the prospects of a mismatch (characterized by a gap or pension hole) in the investment portfolios of such institutions, the author argues.

1.5 Expected Output

The expected outcome of this research study is, therefore, to calibrate a set of Zero-Coupon yield values corresponding to various time duration segments on the basis of Federal Government of Nigeria's Bond (FGN) data series, and plot the same, using appropriate intervals on the Zero-Coupon yield curve. Analysis and interpretation of the Zero-Coupon yield curve shall also be offered, and specific areas of application of the Zero-Coupon yield curve shall, in addition, be extensively explored.

PART I

THEORETICAL FUNDAMENTALS OF THE YIELD CURVE

CHAPTER 2 THE YIELD CURVE AND DEBT FINANCIAL INSTRUMENTS

When it comes to choosing fixed-income financial assets for investment purposes, it is the yield of such instruments, certainly not its price that matters. This is because, as part of the fixed-income products group, debt instruments exhibit fundamentally different characteristics and features to their equity counterparts.

A distinctive feature of fixed income financial instruments is that the magnitude of their coupon payment is pre-determined, *ab-initio*, and will remain fixed throughout maturity period of the debt instruments (in the case of plain vanilla or hard bullet bonds). This explains why the price of such instruments must fluctuate in an opposite direction to interest rates movements to maintain a constant yield or return to investors, according to Fabozzi (2006a), and Papageorgiou (2002).

From an investor's view point, the yield of a fixed-income financial product represents a return on such instrument that perfectly matches or equals the cost of such liability. Approached from another dimension, the yield represents the Internal Rate of Return, otherwise known as the Yield to Maturity or Gross Redemption Yield on the given debt instrument. It is achieved at that point where the net present value of the given investment equals zero. At such a point, it is also known that the market value of such asset is exactly equal to its par value, according to Martellini et.al. (2003), and Fabozzi (2006b).

The yield of a debt financial instrument also serves as a yardstick through which alternative debt instruments of similar features and characteristics can be compared and their profitability ascertained. The yield of a debt instrument also forms the basis for pricing such instrument.

For these reasons, a regular flow of information about yields is highly desirable to fixed-income investors and analysts. To the extent that the risk profile of two debt instruments are identical and exhibit similar characteristics, an investor would opt for a debt instrument that offers a higher return. This assertion applies mainly to hard bullet or plain vanilla bonds; it certainly does not apply

to option-embedded debt instruments, such as callable bonds, puttable bonds, warrants and convertibles, or any other exotic debt instruments. Investors exhibit various motives for locking-in their investments for various maturity lengths. It is, therefore, considered very helpful to offer a set of yields that corresponds to various lengths of investment durations to assist in ascertaining the commensurate market- expected return and compare the same against alternative debt instruments of similar risk profiles and characteristics.

Factually, the yield of a financial debt instrument reflects the annualized percentage increase in the value of the investment and, as such, its curve represents a graphical manifestation of the relationship between the yields on currently tradable debt instruments of the same credit quality, against their term to maturity. The general expectation of financial market participants about the state of interest rate level manifests itself in the level, slope and curvature of the yield curve.

The yield curve effectively serves as an important indicator and a reliable source of information concerning the state of a debt capital market. As such, much of the analysis and pricing activity that takes place in the debt markets revolves around the yield curve appropriate for such debt instrument.

The yield curve offers a set of unique qualities, which makes it one of the most potent tools of financial and macro-economic analysis. Its curve gives a quick, simple and reliable forecasting tool without the rigors of advanced technical capabilities, according to Estrella and Mishkin (1996). It could be used to corroborate or query alternative conclusions obtained from alternative economic indicators. Where a significant discrepancy exists between interpretation of the yield curve and such alternative macro-economic indicators, it would be permissible to question the conclusions from such alternative indicators and uphold the veracity of the yield curve, according to Choudhry (2004).

The yield curve, therefore, qualifies as a good and very reliable indicator of market sentiment, and its shape offers a tremendous amount of high quality information about immediate and future prospects of a nation's economy. Its slope is a reliable prelude to an economic boom or recession, according to Estrella and Trubin (2006), and Estrella et al. (1990). The yield curve has been

known to have captured and adequately reflected the market impact of central banks' adjustments to the base rate, suggests Mishkin (1996).

Each yield curve depicts the cost of capital to a borrower or yield on investment for a lender in that specific market for different durations or maturities and for the specific category of debt issuer to whom such yield curve applies. Unless otherwise indicated, all references to the yield curve in this work shall assume the FGN Zero-Coupon yield curve.

Under normal circumstances, or in an environment devoid of economic crises, Zero-Coupon yield curves form the basis for preparing all other yield curves, and all other debt instruments are priced around government bonds' yield. This is because the Zero-Coupon yield curve is considered to be "risk-free". It also appears to be the most liquid and least risky debt instrument within an economy, *ceteris paribus*. However, the occasional down-grading of the credit rating of sovereign states and the consequential widening of Credit Default Swap (CDS) spreads (especially in moments of crisis) calls this assertion into question, as the recent turbulence in the Euro Zone (particularly with Greece and other PIGS economies) has clearly demonstrated.

All corporate and other yields are inferred on the basis of the Zero-Coupon yield curve by adding a margin, which depicts the risk perception of the obligor, to the government yield in basis points. One basis point is equivalent to one hundredth of one percentage point.

The size of the additional risk premium added on to the Zero-Coupon yield curve varies proportionately and directly with the risk perception of the corporate or municipal obligor, when inferring the corporate or municipal yield, from the Zero-Coupon yield every other thing being equal. Such risk premium is determined on the basis of an independent credit rating of the obligor (the reference entity), the collateral provided, the credit enhancement facilities offered at the moment of securing credit, as well as the liquidity rating of the collateral provided.

In reality, the yield curve is only known with certainty for a very short time interval, usually up to three months. The figures for other time periods or maturities are calibrated by the bootstrapping method and other advanced computational techniques, based on cubic spline mathematics.

It is possible to calibrate yield values on the basis of either bond market data or money market data. Yield curves built on the basis of money market data rely on the prices of "cash" from daily LIBOR rates (or NIBOR, in case of Nigeria) for determining yields of **$t \leq 3$ months**. For yields of **$(3m \leq t \leq 15m)$** , the prices of cash futures are used. For determining the yields of debt instruments of longer maturities **$(1y \leq t \leq 30y)$** the market value of cash swaps is employed.

Where the yield curve is to be built from the bond market data, as is the case with Nigeria, the approach is fundamentally different. For calibrating the yield value on the basis of bond market data, the prices of such bonds must be selected, not from a mixture of different bonds, but rather from a carefully selected basket of bonds with identical characteristics and features. This issue of the homogeneity of coupon basket components shall be explored later and in detail.

Yield curves usually assume asymptotically upward sloping shapes (see Figure 1) and, under a normal and efficient economic environment, would be justifiable. In finance, the arbitrage pricing theory holds that investors, who are willing to forfeit access to their money for lengthier periods of time than their counterparts, shall be exposed to more volatility and economic uncertainties, and, hence, deserve a higher yield premium or compensation for their risk exposure.

During periods of impending economic uncertainty, the yield curve could take opposite direction and be downward sloping, as indicated in Figure 4. This is a manifestation of the fact that the market expects short-term investment to command a higher reward than longer-term investment. This is abnormal, and it as a signal that something synonymous to an economic recession is on the horizon.

2.1 An Overview of Nigeria's Capital and Debt Market

The Nigerian capital market is acutely unbalanced and dangerously skewed in favour of equity products. An overwhelming proportion of Nigerian corporate institutions depend on equity financing to fund their commercial activities, with a tiny minority relying on collateralized loan of one form or another. Over-reliance on equity financing has resulted in the under-capitalization of the Nigerian capital market, inhibiting the nation's economic growth and development.

The 37.5 billion Naira Flour Mills bond that was lead-arranged by the United Bank for Africa (UBA), and completed on Monday 13th of December, 2010 was the largest in sub-Saharan Africa. While this is a commendable stride in the right direction, it shows that a large proportion of Nigerian enterprises are still highly and unduly dependent on loan sharks and equity funds to capitalize their businesses.

The aftermath of the 2007-2009 sub-prime mortgage crises has shown the devastating consequences of over-reliance on limited sources of finance (particularly on equity products) in Nigeria. Equity financing alone does not provide the portfolio diversification required for balanced investment. As a result of over-reliance on equity financing, the entire Nigerian financial system, and her capital market in particular, has been exposed to an unsustainable level of systemic and operational risks.

At the moment, the only visible form of debt instruments in the Nigerian financial system is the Federal Government Bond, which is under the supervision of the Debt Management Office. Other forms of available debt instruments are Commercial Papers and collateralized loans. These alternative debt instruments are largely unregulated, extremely underfunded and traded on the over-the-counter (OTC), thereby posing a significant risk to the Nigerian financial system. A Fully-fledged corporate bond market is likely to take off successfully in Nigeria, following the recent Flour Mill's 37.5 billion naira bond completed debut on December 13, 2010. Appropriate conditions for making this sustainable will be offered in the recommendations to this study.

Debt financing has the potential of complementing the operations of a capital market. It allows investors to diversify and balance their portfolios. To the extent that the local regulation permits, corporate bonds can be structured or customized to meet the investment needs of investors and issuers, and they guarantee a periodic return on investment. Corporate bond comes in a variety of forms – callable bonds, puttable bonds, warrants-linked bonds, convertible bonds, reverse convertibles, fixed/variable coupon bonds, and foreign bonds which come in form of Samurai, Yankee, Matador and Bulldog bonds, etc.

The variety allowed in corporate bonds, as well as the guaranteed return it offers (especially on its plain vanilla version) makes it uniquely attractive and

well sought after in the commercial world. The consequences of weak fixed-income components of the Nigeria's capital markets are summarized below:

2.1.1 Consequences of Weak Fixed-Income Components of Nigeria's Capital Market

- Competitive disadvantage and high cost of borrowing,
- Foreign domination of Nigeria's strategic sectors,
- Very shallow and narrow financial and economic base,
- Limited FDI outflows from Nigeria,
- Nigeria's global financial presence remains insignificant,
- Loss of significant revenue [Singapore, India, Malaysia],
- Underdeveloped technical and manpower base [RSA],
- Low investability and credit rating,
- Relatively weak foreign exchange rate & speculative attacks,
- Undesirable status of a developing economy, and
- Virtual absence of an Energy Bond or an Infrastructure Development Bond to fund electricity, housing and modern health and transportation systems.

Simultaneously, Nigeria's equity counterpart and debt financial instruments suffer from exposure to a number of risk factors.

The common risk factors to which corporate bond and other forms of debt instrument are exposed include: credit risk, reinvestment risk, interest risk, operational risk, systemic, country-specific risk, duration risk, inflation risk, global-specific risk, etc. Investors and practitioners need not be oblivious of such risks. They should also be knowledgeable about, and possess practical skills regarding, non-derivative as well as derivative products, including futures, options, swaps and swap options, Credit Default Risk (CDS), etc, with which to hedge and mitigate such risk factors.

The prerequisites for successful introduction of fixed-income financial instruments into Nigeria's capital markets are summarized as follows:

- A sound insolvency, securitization and related legal infrastructure;
- A coherent macro-economic policy (interest rate; exchange rate; inflation tax);
- A good country-specific and credit rating;
- Effective debt and risk management techniques; and

- A good technical and manpower base.

The Central Bank of Nigeria is at the fore-front of providing the platform and other necessary pre-requisites for corporate bonds to take off and complement the equity financing instruments within the Nigerian financial system. One good step in this direction is the initiative to commission the project to calibrate and fit the Nigerian Government yield curve. This will form the basis for deriving other necessary yield curves, including the corporate bond yield curve, the annuity yield curve, the money market yield curve and other yield curves required to strengthen the Nigerian financial market.

CHAPTER 3 THEORETICAL FRAMEWORK OF THE YIELD CURVE

According to Brown (1994), the behaviour of the yield curve is guided by a set of economic theories, which provide technical explanations to support the shape, slope and time movements of the curve. Prominent among such theories are the liquidity premium theory, the market expectation theory, the market segmentation theory and the preferred habitat theory.

3.1 The Market Expectation Theory

This theory anticipates a moderate increase in interest rates over time. The theory further holds that the risk premium of investors increases with maturity in a decreasing proportion, according to Barrett (1995) and Choudhry (2004).

The positively-sloping yield curve holds that longer-dated debt instruments should generate, on average, higher yields over an investment period. This notion can be expressed in mathematical expression such that:

$$L_1 < L_2 < L_3 < \dots < L_n$$

and

$$(L_2 - L_1) > (L_3 - L_2) > \dots > (L_n - L_{n-1})$$

Where L represents the premium for a bond with term to maturity for n years. The above expressions firmly indicate that a normal yield curve will have a positively-sloping curve, with the degree of slope steadily decreasing as the maturity of the bond lengthens. This view is illustrated diagrammatically in Figure 1.

3.2 The Liquidity Premium Theory

This theory asserts that investors expect a premium or compensation for holding long-term debt instruments and, impliedly, for providing liquidity to the market. The magnitude of such liquidity premium decreases with its duration and is dependent, among others, upon the size of the debt instrument and the investment duration to which it is subjected.

Its component, among others, are made up of a liquidity risk premium (compensation for having investors' savings tied-up over a period of time), a credit risk premium (compensation for increased risk of default) and a volatility risk premium (compensation for the uncertainty of future return or earning).

This theory expects that long-dated bonds attract more yield than short-dated bonds. As such, it corroborates the upward-sloping shape of the normal yield curve, and further substantiates the market expectation theory.

One severe limitation of this theory, however, is its inadequacy in explaining the negative and humped slopes, as well as its ignorance of reinvestment risk.

3.3 The Market Segmentation Theory

This is another theoretical foundation upon which the behaviour of the normal yield curve and its shape rests. Provided that no significant arbitrage opportunity exists, this theory affirms that rates on long-dated, fixed-income instruments can be expressed as the geometric mean of the yields on a series of short-dated, fixed-income products.

This theory further holds that there exists different category of investors, and each invests according to its liabilities, and, as such, remains firmly confined to, or operate within, a given segment on the yield curve. Two major players in the fixed-income investment universe, who acutely observe this investment principle, are commercial banks on one hand, and pension funds and life insurance companies on the other.

Life insurance companies and pension funds are structural buyers of long-dated debt instruments because they hold long-dated liabilities on their balance sheets. These two institutional investors strive constantly to maintain balance and avoid a mismatch between the values of their assets and those of their liabilities. As such, they operate overwhelmingly in the long-dated segment of the yield curve.

Commercial banks and their Nigerian counterparts, on the other hand, are acutely volatile in their investment attitude. Whilst they often exhibit a cyclical investment pattern, they remain mainly confined to the short-end of the yield curve. They feel comfortable operating in the short-end of the yield curve because they hold large amounts of sight deposits in their liability portfolios. This compels them to invest in short-dated debt instruments and hold near-liquid assets. For this reason, commercial banks are major players in 3, 6 and 12 month-dated bonds, as well as money markets products using LIBOR or NIBOR rates. Trading in short-dated instruments enables commercial banks to guarantee the liquidity level required by their commercial activities.

Owing to the investment capacity of the above-mentioned short and long-dated participants, their investment activities, collectively, determine the shape of the yield curve to a very large extent and the same pattern is true for

Nigeria. The above view is in line with the postulation that commercial banks traditionally borrow short, and invest short.

It must, however, be noted that the 2007-2009 global financial crises have thrown this long-standing paradigm into considerable doubt. Sholarin (2009) (a & b) contends that the 2007-2009 global financial crisis was triggered partly as a result of commercial banks switching from their traditional investment pattern of short-term borrowing and short-term lending to short-term borrowing and long-term lending (in order to benefit from the securitization and CDO activities on sub-prime mortgage loans) thereby exposing them to a severe liquidity crisis.

3.4 The Preferred Habitat Theory

This theory rests on two premises. First, the theory holds that, judging by the concave shape of the yield curve, and except where the yield curve is flat (as in Figure 5), the rate of growth of the risk premium is not constant (see Figure 1 and Figure 4). Regardless of this non-uniformity, investors prefer to remain firmly in their segment of the yield curve, in line with the market segmentation theory and as dictated by their future liability requirements. Secondly, this theory affirms that investors would not all at the same time terminate their position and alter their investment pattern in the same way and manner as profit opportunities arise in the market. This is because some investors would, again, prefer to remain in their yield curve segment and trading their usual short, medium or long-dated bonds. This might partly be due to legal constraints, or in line with the future liability requirements of participants.

The theory, nevertheless, holds steadfast on the view that whenever a considerable arbitrageur opportunity persists in the market, some lenders and borrowers will abandon their preferred habitat on the yield curve - albeit temporarily - to take advantage of such unusual opportunities in other segments on the yield curve, provided the benefit of doing so outweighs the cost. This notion corroborates the view that all segments of the yield curve are actively traded and remain functional at all times.

PART II

MODELING NIGERIA'S ZERO-COUPON YIELD CURVE

CHAPTER 4 RELEVANCE OF THE YIELD CURVE TO NIGERIA'S FINANCIAL SYSTEM

Yields on Nigeria's debt instruments avail investors the opportunity of ascertaining the relative value of debt investment opportunities in Nigeria. The Nigeria's Zero-Coupon yield curve represents an important indicator, as well as an invaluable knowledge base, about the state of the country's debt market. It shows the market's anticipated returns on debt instruments for a variety of maturity dates. In the same manner, the yield curve summarizes the risk-free cost of credit or loans of various maturities within Nigeria. Effectively, it represents an indicator through which the confidence of debt investors in Nigeria, and, to a large extent, the availability of growth capital in Nigeria, could be measured. In this regard, the shape that the yield curve exhibits has been known to have been used in predicting future patterns of economic growth or recession (See Estrella, 2005; Papageorgiou, et al., 2002 and Watson, 1991).

Nigeria's Zero-Coupon yield curve represents the yields on risk-free lending, and it forms the basis for calibrating other yields. The shape of this default-free yield curve is, therefore, of special interest to practitioners in any financial market, according to Wu (2003) and Ilmanen (1997). Nigeria's yield curve represents a snapshot of the current level of yields as perceived by her market. As such, Nigeria's Zero-Coupon yield curve is forward-looking; it does not capture the historical pattern of such yields. For this purpose, according to Choudhry (2004), it is important that Nigeria's yield curves be prepared as frequently as possible.

The unique roles and relevance of debt instruments within Nigeria's capital market, as well as on the wellbeing of her overall economy, cannot be overemphasized. The depth and breadth of Nigeria's capital markets, in terms of its capitalization level, its variety as well as its level of sophistication in trade implementation, represents a measure of the extent of maturity and advancement of her financial system.

Strengthening the debt markets in Nigeria (especially its corporate sector element), and allowing their associated derivatives complements (including

securitization or debt warehousing) to flourish, can generate an enormous amount of capital to transform the country's infrastructure and set the country on a course to becoming a financial hub in emerging markets.

The Federal Government of Nigeria's Zero-Coupon Yield Curve provides a yardstick through which both local and foreign investors can precisely assess the viability of Nigeria's debt markets, and, hence, its investability over a wide spectrum of maturity or duration.

It must be emphasized, however, that, given the level of sophistication of her financial markets, Nigeria will need more than a robust and well calibrated yield curve to attract investors into her financial markets. It is equally important to complement such yield curve with sound legal infrastructures, efficient and well-functioning financial markets, as well as very prudent and well coordinated macro-economic policies. Other essential co-requisites are: an army of highly skilled and experienced financial experts, as well as an operationally efficient clearing house for debt instruments. Internationally recognized credit rating agencies, as well as an opportunity to pool, tranche, rate and securitize such debt instruments for onward transfer to debt investors at spot, swap, futures and forward markets will equally be required. These will form the basis of this paper's recommendations.

In order to ensure a successful integration of debt instruments into the Nigeria's financial system, it is appropriate to pay attention to the issue of onward securitization of Nigeria's debt instruments, right from their inception stage, as the need for this will arrive sooner than expected, in view of Nigeria's economic and financial landscape and her potential.

However, highly exotic and unstable structured financial instruments (such as Collateralized Debt Obligations (CDOs) should be handled with extreme caution and should be well regulated and reviewed periodically. While structured financial instruments have huge potential for generating supernormal returns if handled by capable hands, they can equally turn into a financial "tsunami" if handled carelessly. Sound regulation and periodical monitoring by the regulators will be indispensable in achieving such outcomes according to Sholarin (2009a, b).

Availability of the recommended ancillary facilities would impact directly on the types of instruments tradable on the Nigerian Stock Exchange, the

frequency of transactions' as well as the size or magnitude of such transactions. This, undoubtedly, would make Nigeria's debt, as well other financial-instruments very attractive to a wide variety of investors, especially at the international arena.

If properly implemented, an enhanced and well-functioning Nigerian debt market has the potential of attracting far more capital from inside and outside the country than what the capitalization of her equity-based instruments currently offers, and certainly more than enough capital to transform Nigeria's economy and infrastructure.

To the extent that data availability permits, this research work seeks to calibrate FGN's Yield Curve, highlight its monthly variations, and plot its Zero-Coupon yield curve. The Zero-Coupon Yield Curve so generated will then be interpreted, analyzed and benchmarked against that of other reference countries (including South Africa, Britain, India, Greece and Germany, etc). The intention here is to capture intra-yield and inter-yield spreads, as well as differentials in curvature, levels and slopes, which might exist among the pre-selected yield curves.

While a variety of yields do exist in the financial markets and serve various purposes (for example current yield, yield to call, yield to put, forward yield and yield to maturity), this research paper focuses mainly on calibrating Nigeria's yield to maturity, henceforth referred to as "the yield to model and fit Nigeria's (FGN) Zero-Coupon yield curve." Nigeria's forward yield shall be inferred from the FGN Zero-Coupon yield curve, fitted and equally analyzed.

The forward yield is needed to facilitate the predictive power of the country's FGN Zero-Coupon yield curve in forecasting short-term interest rates – an area of special interest to the Monetary Policy Committee of the Central Bank of Nigeria (CBN).

4.1 Key Steps to Deriving Nigeria's Zero Coupon Yield Curve

4.1.1 Preparations for Calibrating Nigeria's Zero-Coupon Yield Value

The starting point for modeling yield curves for any country, including Nigeria, is to select a coherent basket of "identical" bonds from which the yields will be extracted, modeled and fitted. Depending on the needs of the issuer and the buyer, bond markets constantly offer a wide variety of products

engineered and structured to meet the investment needs and risk appetites of various investors. As such, the yield of such bond instruments cannot be identical; they would be commensurate with the level of risks applicable, as well as the prevailing market conditions. For this purpose, it is crucial that only bonds of similar characteristics are used when extracting their yield. This would facilitate comparison of bonds, as well as their technical analysis. Two synchronization steps must be taken as a prelude to calibrating Nigeria's FGN Zero-Coupon yield value: Synchronization of information about the issuer, and synchronization of information about the issue.

4.1.2 Synchronization of Information about the Issuer

All bonds to be included in modeling yield curves for Nigeria must be identical, in the sense that such bonds must be issued only by the Nigerian Federal Government. As such, bonds issued by state and local governments, or state-owned entities (e.g., NNPC, FMBN, PHCN, etc) or any form of their combinations shall not be considered.

Additionally, the bonds to be used must be issued from one and the same country. For example, it is possible for Nigerian Federal Government to raise capital abroad by issuing international or Eurobonds outside Nigeria. For the purpose of modeling Nigeria's yield curve, such bonds shall not be considered.

The international rating assigned to the bonds shall also be identical. Nigerian government bonds are often rated by local rating agencies (currently dominated by Augusto). As the intention here is to model yield curves that match international standards, we shall compare the outcomes of Augusto's rating of Nigerian government bonds with those issued by Moody's, Fitch or Standard and Poor for bonds of similar quality, and where discrepancies exist, preference shall be accorded the rating outcomes of international rating agencies.

4.1.3 Synchronization of Information about the Issue

As the Nigerian Federal Government offers a wide variety of bond issues on a regular basis, selection from among such bonds for the purpose of calibrating and modeling Nigeria's Zero-Coupon yield curve shall be performed, bearing in mind the following features:

Optionability of the instrument (this refers to the callability, putability and convertibility of the bond). The issue of bond optionability plays a crucial role in selecting the bonds to be included in the common basket. This is because inclusion of optionable bonds (where the issuer/buyer reserves the right to put, call or convert bonds) alongside non-optionable bonds in a basket of bonds for the purpose of yield value calibration, can significantly alter the market price of such bonds and fundamentally distort the homogeneity of the entire bond basket, especially the risk elements and implied risk-adjusted yield. This makes comparison of such diverse categories of bonds questionable. For the purpose of modelling yield curves for Nigeria, only non-optionable bonds will be considered. To do otherwise would render the outcome of the exercise technically spurious.

Furthermore, all bonds for inclusion in our reference bond basket for Nigeria must be identical in terms of: amount issued, amount outstanding, issue date, issue price, maturity date, bond's currency denomination, type of coupon, coupon frequency, coupon rate, day-count basis (actual/actual; actual/365; actual/360; 30/360), redemption value, and daily trading volume of the bond, among others. The above features are exhaustively explained in Fabozzi (2006 a, b), Martellini, et al. (2003) and Choudhry (2004).

The essence of using the features listed above for selecting the basket of bonds to be used in calibrating Nigeria's yield curve is to achieve two objectives. One is to ensure price homogeneity. This eliminates pricing anomaly as only bonds held to maturity will be considered, and only their Yield to Maturity will have to be calibrated.

The Nigerian financial system has not reached the level of sophistication where futures, forward, options, swaps, swap option and other exotic derivative instruments might be widely traded, although some over-the-counter (OTC) activities have recently been uncovered. As such, yield to put and yield to call shall not be calibrated for Nigeria until, perhaps, after five years, provided the present level of advancements is sustained.

The second objective of selecting a reference bond basket on the basis of the features mentioned above is to avoid illiquidity or over liquidity among our reference basket of bonds. Illiquid bonds will not be considered in our reference basket bonds for Nigeria's zero-coupon yield curve because such bonds tend to be inherently underpriced as buyers often have a liquidity

premium built into their prices. In the same manner some bonds may be over liquid, and will not be considered in our reference basket of bonds for Nigeria because of the possibility of being overpriced, with a liquidity premium or "haircut" added to the price. It must be mentioned, however, that assessing the degree of liquidity of a given bond poses enormous practical challenges, and its consideration is beyond the scope of this paper. Notable among the authors, whose works address the issue of liquidity is Ameida et.al, (1998), Ladic et.al, (2003) and Fabozzi (2006a).

From my discussions with market practitioners and those involved in calibrating the yield curves for England, Germany and South Africa, this research uncovers that the common practice, when it comes to an effective reference for bond basket selection, is to take into account the size of the issue, the nature of the issue (on-the-run or off-the-run) and the daily traded volume of the bond. Adhering to the condition maintained above would ensure that the bonds being considered for the basket exhibit similar characteristics and are like-for-like in nature.

The author acknowledges the usefulness of JP Morgan's monthly *Global Bond Index*, where the liquidity states of the traded bonds are given considerable attention. The Index has proved an invaluable source of knowledge in preparing this report.

Assuming that there are enough bonds to cover the entire maturity spectrum, the bond basket for calibrating Nigeria's Zero-Coupon yield value must, in addition, contain a mix of "active" bonds (with a significant daily turnover) and "traded" bonds (with prices that fluctuate regularly, and for which two-way markets exist).

4.2 Alternative Yield Estimating Techniques: How Good Are They?

Conventionally, a yield curve can be estimated on the basis of either the discount factor function, or discreet yields of the theoretical zero-coupon (spot) obtainable from the yield to maturity of a pre-selected basket of coupon-based bonds (Choudhry, 2004).

Using the discount function approach to estimate the yield of a debt instrument has a number of technical faults. Firstly, the method used to derive the discount function relies on more than one price source (mainly from the money market, the futures market and the Swaps market) to produce a

combined and integrated yield curve. Mathematically, it is almost impossible to integrate such separate functions into one unified curve without leaving undesirable “knots” or “kinks” along the curve.

Another limitation of using the discount factor function to model the yield value for Nigeria is the fact that some of the key components considered in creating separate yield values (especially the money market and, to a lesser extent, swap data sets, incorporate credit risk premiums, which represent the default risk inherent among the interbank market counterparties, according to James and Webber (2000).

A further drawback of using swap-based data sets in calibrating the yield value is the fact that small oscillations in short-term rates cause disproportionate variations in long-dated spot rates, and even wider variations in the forward rates. This makes using swap data sets less accurate in estimating the yield value.

Another limitation of using the discount factor function to model the yield curve is the fact that futures rates, which are the other major elements of the discount factor approach, do not reflect default risk. As an exchange-traded instrument, the exchange clearing house assumes full responsibility for the counterparty risk for futures transactions concluded on its trading platform. Another factor which limits reliance on futures rates in calibrating the yield value, is the fact that future rates are usually treated as one-point period rates, which effectively equate them to the Forward Rates Agreement (FRA), the cash flow of which is quoted at a discount. This condition is in contrast to the futures trades' contract, which requires daily margin calls. For this purpose, a convexity adjustment is usually required to accurately convert futures trades to FRA rates and make them useful for yield value calibration.

The above arguments build the case in support of the theoretical zero-coupon (spot) rate as a better alternative than the discount factor approach, when it comes to selecting alternative yield estimating techniques for Nigeria.

By using the theoretical zero-coupon (spot) rate yields to model yield values for Nigeria, it is possible to derive reliable and robust implied forward rate values for Nigeria. This would be invaluable in later years for hedging purposes, when the use of derivatives instruments becomes fully functional and mature in Nigeria's financial markets.

Another unique benefit of using the theoretical zero-coupon (spot) rate to estimate Nigeria's yield value is its unique benefit of serving as the starting point from which other debt instruments' yield values (such as corporate bond yields) can be inferred. The usefulness of this in strengthening the corporate bond market for Nigeria could be substantial.

After estimating the yield value of zero coupon bonds, then the yield curve itself can be fitted using one of the following methods:

- Linear interpolation,
- Logarithmic interpolation,
- Polynomial models [Here the order of the polynomial determines the shape of the curve],
- Cubic splines, or
- Regression models.

The most appropriate curve fitting method in the case of Nigeria would be the cubic splines-based method. Using alternative methods, such as the parametric regression models, is not ruled out as the conditions become perfect and data become readily available in the future. These alternative curve fitting methods are discussed extensively in major applied mathematics texts.

4.3 Modeling an Implied Zero-Coupon Yield Value for Nigeria

The task of calibrating the zero-coupon yield value involves inferring the price of zero-coupon bonds from the price of a set of coupon-bearing bonds.

However, this investigation reveals that Zero-Coupon bond prices are not readily available in Nigeria. This is because such products are not traded on a daily basis, and, where such prices are available, the products are not traded in sufficient quantities. In line with the market convention, this research paper infers the prices of such zero-coupon bonds from the prices of coupon-traded bonds, and subsequently derives the zero-coupon yield from such zero-coupon prices. As the reference bond basket selected for Nigeria contains annual zero-coupon yields, this research sees no need to annualize such inputs before using them in its calibration. When Nigeria changes from issuing annual coupons into a bi-annual coupons base, then there would be the need to annualize such bi-annual coupons before using them as inputs in yield calibration. The essence of this, according to Choudhry (2004), is to ensure

that the required homogeneity of essential inputs for the yield calibration is maintained.

One of the fundamental principles of pricing fixed income instruments is the fact that most coupon-bearing bonds can be technically viewed as a portfolio of zero-coupon securities. This inference, however, comes with a caveat, that it remains valid so long as there is no credit risk, and no options are held by the bond issuers to call or by investors to put or to convert the bonds.

Inferring FGN Zero-Coupon prices from its coupon-based bonds is extremely challenging, and involves precise application of bootstrapping techniques. This is because there are some maturity dates for which no issues are present in the market. Worse still, some maturity segments for the FGN coupon-based bonds are spanned by issues that exhibit characteristics consistent with convertible bonds, which render such inputs useless for the purpose of zero-coupon yield calibration. In line with the standard practice, relying on very sound benchmarks at each maturity date, bootstrapped-based techniques are applied to infer estimates of yields at those maturities and then cubic spline mathematics is applied to fit the curves (see Martellini, et al., 2003). It is this art of assigning yields to every maturity date that underlies the concept of the term structure of interest rates.

The major inputs for calibrating periodical yields (i.e. the term structure of interest rates) for Nigeria comprise the following debt structures:

1. Nigerian Treasury Bills
 - i. 91 -day
 - ii. 182-day
 - iii. 365-day

2. FGN Bonds
 - i. 3-year
 - ii. 5-year
 - iii. 7-year
 - iv. 10-year
 - v. 20-year

The Central Bank of Nigeria (CBN) offers Treasury Bills to mop up money supply, as well as curb the inflation rate in a series of 91-day and, occasionally, 182-day and 364-day bills. The issuance and management of FGN or treasury

bonds is restricted almost entirely to the Debt Management Office, in conjunction with the Central Bank of Nigeria.

Highlights of the procedures for inferring zero-coupon bonds' prices and yields on the basis of coupon-bearing bonds, using bootstrapping techniques, are illustrated below:

Given that P_1 is the price of bond (1), and C_1 denotes the annualized Naira coupon associated with such bond, then the price of such bond (1) can be represented by:

$$P_1 = \frac{100+C_1}{(1+Y_1)} \quad (1)$$

Where Y_1 represents the one-year spot interest rate.

Substituting for C_1 and P_1 in equation (1), it is possible to obtain the value of Y_1 from which it is possible to obtain the one-year implied zero coupon price as:

$$Z_1 = \frac{1}{1+Y_1} \quad (2)$$

As such, once the C_1 (coupon) and P_1 (price) of a one-year, coupon-based bond are known, then it is possible to find the price of the Zero-coupon equivalent.

Knowing the value of Y_1 makes it easy to determine its Y_2 equivalent, using the substitution method. As such,

$$P_2 = \frac{C_2}{(1+Y_1)} + \frac{100+C_2}{(1+Y_2)^2} \quad (3)$$

Given the values of C_2, P_2 and Y_1 , it is possible to solve for Y_2 . The implied zero-coupon price for year two zero-coupon bond can then be found by:

$$Z_2 = \frac{1}{(1+Y_2)^2} \quad (4)$$

From the above finding, it follows that given the values of C_2, P_2 and Y_1 , it is possible to solve for Y_2 . In the same manner if the values C_2, P_2, Y_1 and Y_2 are given, it is possible to ascertain Y_3 , and subsequently Z_3 as follows:

Given that, the values of P_3, C_3, Y_1 and Y_2 are known, then finding Y_3 requires substituting for Y_3 in the equation:

$$P_3 = \frac{C_3}{(1+Y_1)} + \frac{C_3}{(1+Y_2)^2} + \frac{100+C_3}{(1+Y_3)^3} \quad (5)$$

Where the amount of coupon settlements is constant and frequent, it is possible to estimate $Y_3 \dots \dots Y_n$ using a standard formula:

$$Y_n = \left[\frac{\frac{C}{2} + 100}{P_n - \frac{C}{2} \sum_{t=1}^{n-1} \frac{1}{(1+\frac{1}{2}r_1)^t}} \right]^{\frac{1}{n}} - 1 \quad (6)$$

Please note that a bi-annual coupon payment frequency is assumed in the equation (6) above.

Having found Y_3 from equation (5 or 6) above, the implied zero-coupon price for year three zero-coupon bond then becomes,

$$Z_3 = \frac{1}{(1+Y_3)^3} \quad (7)$$

A Note on Exponent "N"

Equations (1-7) above assume that coupon settlement dates coincide with the dates of issue. However, where the bonds are purchased between coupon payments with "n" coupon payments remaining, then equation (5) will be equivalent to:

$$P = \frac{C_1}{(1+i)^w} + \frac{C_2}{(1+i)^{1+w}} + \frac{C_3}{(1+i)^{2+w}} + \dots \dots \dots \frac{C_N}{(1+i)^{n-1+w}} + \frac{M}{(1+i)^{n-1+w}} \quad (8)$$

In Nigeria, the settlement date and coupon date vary significantly in both positive and negative ends of the duration spectrum and are in line with the technical and data constraints inherent in Nigeria's financial landscape. This forces the accrued interest to tend to cancel out one another. For this reason, the accrued interest shall be ignored in calibrating the price of FGN bonds, and equation (5) shall be preferred in lieu of equation (8) in our pricing equation for FGN bonds. Equation (8) shall, nonetheless, be invaluable, for example, in pricing corporate bonds and other debt instruments where

reliable data representing the time interval between the coupon and settlement dates is readily available.

Since the semiannual coupon payments, in this circumstance, are equivalent to an ordinary annuity, then the present value of:

$$\frac{C_1}{(1+i)^1} + \frac{C_2}{(1+i)^2} + \frac{C_3}{(1+i)^3} + \frac{C_N}{(1+i)^N}$$

Can be expressed as:

$$C \left[\frac{1 - \left[\frac{1}{(1+i)^N} \right]}{i} \right] \quad (9)$$

This is the same as the formula for the present value of an ordinary annuity, according to Fabozzi's fixed income mathematics in Fabozzi (2006b).

From this, equation (8) then becomes equivalent to:

$$P = C \left[\frac{1 - \left[\frac{1}{(1+i)^N} \right]}{i} \right] + \frac{M}{(1+i)^N} \quad (10)$$

For a zero-coupon bond, such as that of the FGN series, the price will (without the coupon) be:

$$P = M \left[\frac{1}{(1+i)^N} \right] \quad (11)$$

The issue of n becomes very important, especially where coupon payment dates are different from settlement dates, and the exact number of dates for which a coupon accrues need to be accurately determined. This is necessary to ascertain how many days must lapse before the next coupon date, and to address the issue of how to determine the present value of cash flows received over fractional periods. Another reason for this is to ascertain how much the buyer must compensate the seller for the coupon interest earned by the seller for the fraction of the period that the bond was held. For this and other purposes, the issue of "day count" basis plays a very crucial role in the arithmetic of bonds, according to Fabozzi (2006b).

In the specific case of Nigeria, the day count basis can be useful for the purpose of ascertaining:

- Accrued interest payable on a bond;
- Next coupon payment;
- The exponent used to compute the present value of the cash flow from the next coupon date, back to the settlement date;
- The coupon payment on the maturity date, and
- The exponent used to compute the present value of the cash flow at the maturity date back to the last coupon date.

While a number of countries, such as the EMU countries, use the Actual/Actual, South Africa and Nigeria use the Actual/365.

It must be emphasized that from equations (1-11), it can be deduced that the price of an instrument (in this case a bond) changes in an opposite direction to its yield in accordance with the conventional wisdom in fixed income mathematics.

Equations 1-11 illustrate the concept of deriving the zero-coupon yield from coupon-based bonds. A generalized equation for deriving zero-coupon bond prices from a set of coupon-based bond prices can be found by using matrix algebra where the cash flow (of the coupon and principal) of a given set of bonds will have to be inverted and multiplied by the price vector of the coupon-based bonds (Martellini et al. (2003) and Choudhry (2004)).

It must be noted that the zero-coupon prices derived in this way do not represent real market values. Rather, they implied zero-coupon bond prices, consistent with the set of market prices from which we can then extract annual compounded zero-coupon rates at date (t) and maturity (n).

Our illustrations, however, presuppose that bonds are available at every maturity sector on the (X) axis of the yield curve. In reality this can be very challenging in the sense that some maturity dates might be populated by callable, puttable or convertible bonds, which are less useful in calibrating the zero-coupon yield curve.

The most feasible alternative to overcoming this limitation is to use the bootstrapping method to populate those empty maturity dates on the (X) axis

with implied zero-coupon yields, using the procedures illustrated in equations 1-7.

The yield value for FGN Zero-Coupon bond for the month of June 2010, calibrated in accordance with the procedures discussed above and on the basis of which various segments of the FGN Zero-Coupon shall be fitted, is hereby presented in Table 1 for your perusal. Similar yields for the month of May 2010 are offered in Table 2.

**Table 1: The FGN Yield Value for the Month of June 2010
Auction**

Issue date	Tenor	R/Y	Coupon	Maturity	Price	TTM
17-Jun-10	91-Days	2.52	2,50	17-Sep-00	99.97	0,25
17-Jun-10	182-Days	3.64	3,51	17-Dec-00	99.94	0.50
10-Jun-10	364-Days	4.06	3,90	10-Jun-11	99.85	1.00
25-Jun-10	3-year	6.24	5.50	19-Feb-13	98.21	2.65
25-Jun-10	5-year	7.10	4.00	23-Apr-15	85.63	4.83
23-Jun-10	10-year	7.13	7.00	23-Oct-19	107.21	9.59
25-Jun-10	20-year	8.69	8.50	20-Nov-29	98.22	19.4

Table 2: The FGN Yield Value for the Month of May 2010

Auction						
Issue date	Tenor	R/Y	Coupon	Maturity	Price	TTM
27-May-10	91-days	1.61	1.6	27-Aug-10	100	0.25
27-May-10	182-days	3.1	3.2	27-Nov-10	100	0.5
13-May-10	364-days	4.12	3.99	13-May-11	99.88	1
21-May-10	3-year	6.42	5.50	19-Feb-13	93.34	2.74
21-May-10	5-year	6.89	4.00	23-Apr-15	80.45	4.92
26-May-10	10-year	7.13	7.00	23-Oct-19	107.21	9.59
21-May-10	20-year	10.00	8.50	20-Nov-29	87.24	19.5

PART III

FITTING AND SMOOTHENING NIGERIA'S ZERO-COUPON BOND

CHAPTER 5 FITTING NIGERIA'S ZERO-COUPON YIELD VALUE

After calibrating the yields for various maturity dates, the next step is to embark on fitting the yield. According to James and Webber (2000), curve-fitting techniques fall into two categories: the parametric method, and the Spline-based method. The parametric method is extensively explored in the work of Nelson and Siegel (1987) and Anderson and Sleath (1999), while Waggoner (1997) adequately investigates the Spline-based approach. A good example of the parametric model is the Vasicek model.

Institutions, such as the Bank of England, use a modified version of the Svensson yield curve model, which belongs to the family of the one-dimensional parametric yield curve model. This is identical to the Nelson and Siegel yield curve model, which is particularly useful in modeling the forward rate yield curve. The constraints of data availability and technical feasibility suggest that the Spline-based curve-fitting method is the most appropriate for fitting Nigeria's yield curve.

The cubic Spline method fits different polynomials over different, but overlapping, terms to maturity. The individual curves (vertices) so produced (see Figures 1, 2, and 3) are then fitted together, by applying a unique cubic equation between the vertices, to produce a single smooth curve over the entire term structure. The outcome of this is a chain of independent cubic equations bonded together into one single cubic curve, as illustrated in Figure 4.

According to James and Webber (2000) and Choudhry et al. (2001), the cubic Spline method helps to preserve the smoothness of the separate vertices. It also adds some degree of stiffness to the combined curve. However, it must be used carefully and with a reasonable level of mathematical intuition. From Table 2, we obtain the identical version of Figure 4 for the month of May 2010 (see Figure 5). Figure 6 compares the yield curves of Nigeria for May and June 2010 and attempts to highlight the inherent spreads contained therein.

The relevance of this in monitoring the 30-day market-expected monetary policy forecast for the Monetary Policy Committee of the Central Bank of Nigeria is highly significant.

One reliable measure of authenticity and reliability of a yield curve is that, in theory, it must present no arbitrage opportunity along its curve. What this means is that, using interest rates available today, the return from buying a two-year bond must be equal to the return from buying the one-year bond, and rolling over (or reinvesting) the proceeds for another year. This is in line with the fundamental principle of the break-even principle, or the law of no arbitrage.

The same principle forms the fundamentals against which forward rates are implied. The caveats surrounding this assertion, however, must not be lost on any one, i.e., the reinvestment rate as well as the risk premium must remain constant.

Using the short segment (90 days to 364 days Nigerian Treasury Bills), the medium segment (364 days to 5 years FGN Bonds) and the long segment (5 years to 20 years FGN bond) for the month of June 2010 (see Table 1), we identify a set of three cubic equations $\vartheta\mu_{(i,t)}$ with each one connecting two adjacent vertices n_i and n_{i+1} in the following manner:

$$\vartheta\mu_{(0,t)} = a_0n^3 + b_0n^2 + c_0n + d_0, \text{ Connecting vertex } n_0 \text{ with } n_1 \text{ which spans 90 days – 1 year vertices (see Figure 1),} \quad (12)$$

$$\vartheta\mu_{(1,t)} = a_1n^3 + b_1n^2 + c_1n + d_1, \text{ Connecting vertex } n_1 \text{ with } n_2, \text{ this spans 1 years – 5years vertices and (see Figure 2)} \quad (13)$$

$$\vartheta\mu_{(2,t)} = a_2n^3 + b_2n^2 + c_2n + d_2, \text{ Connecting vertex } n_2 \text{ with } n_3, \text{ this spans 5 years to 20 years maturity interval (See Figure 3)} \quad (14)$$

In each of equations (12)-(14), coefficients a, b, c, and d are the unknowns. We shall then require a total of twelve conditions in order to solve the entire system of equations (12), (13), and (14).

Figure 1: Vertice 1 - Segment of June 2010 FGN YC

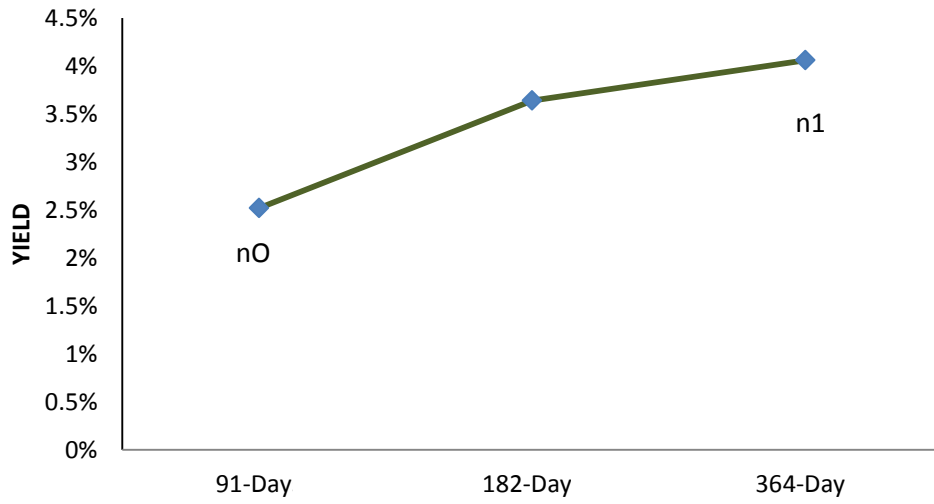


Figure 2: Vertice 2 - Segment of June 2010 FGN YC

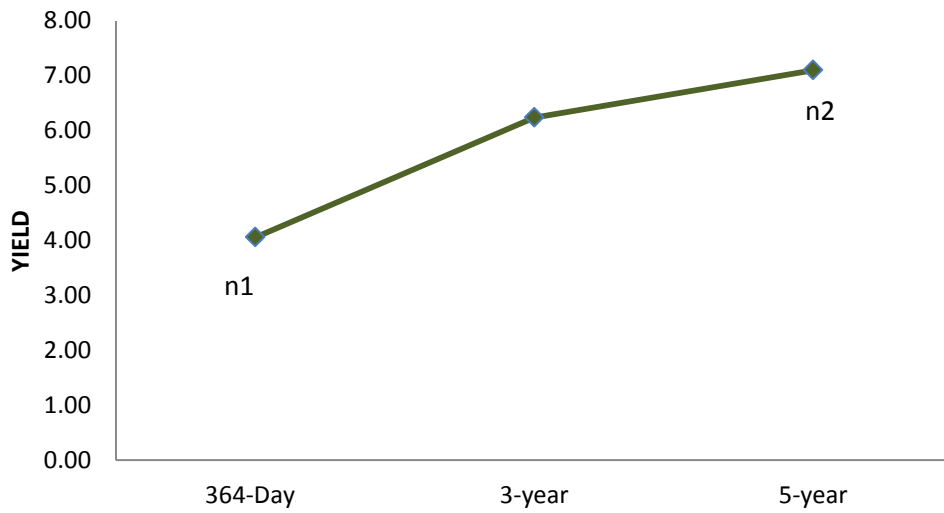


Figure 3: Vertice 3 - Segment of June 2010 FGN YC

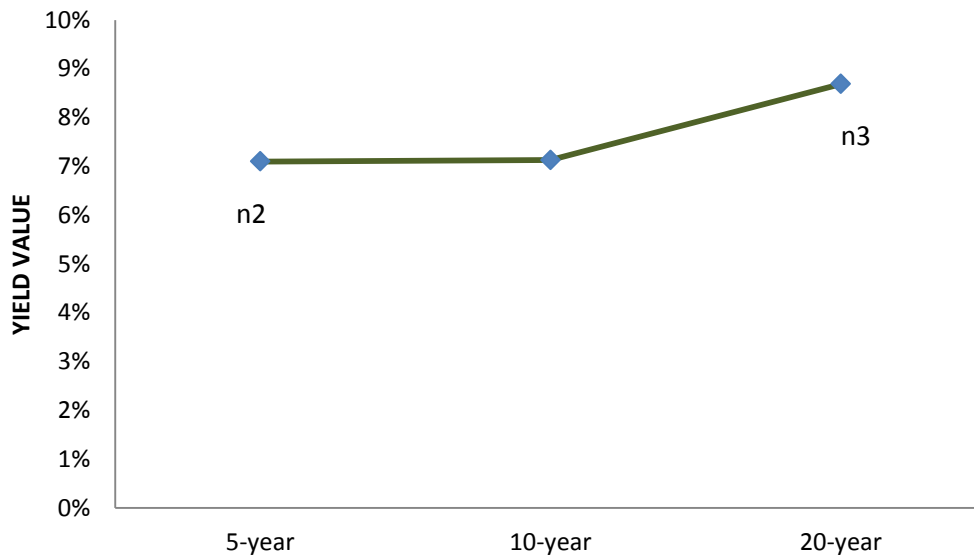


Figure 4: FGN Yield Curve as at June 2010

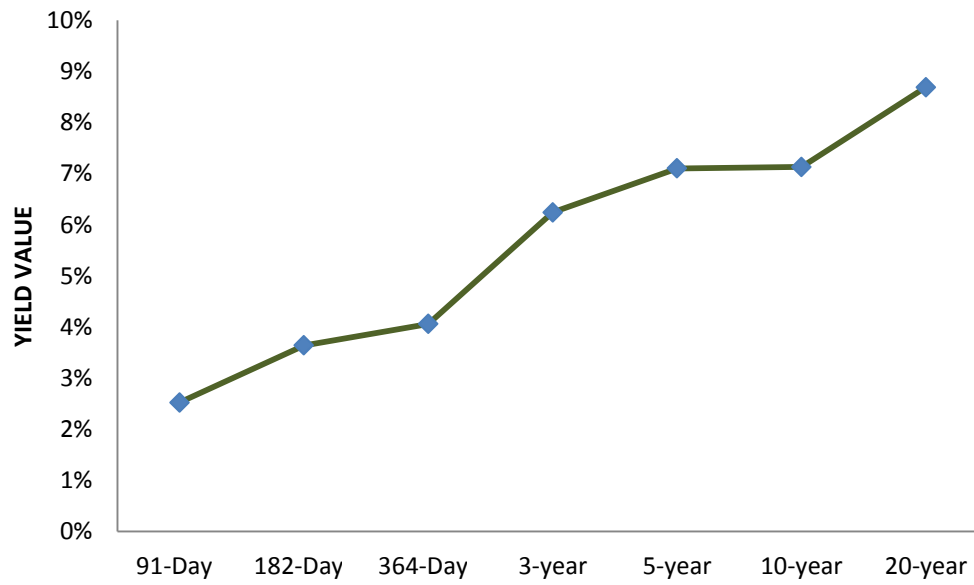


Figure 5: FGN Yield Curve as at May 2010

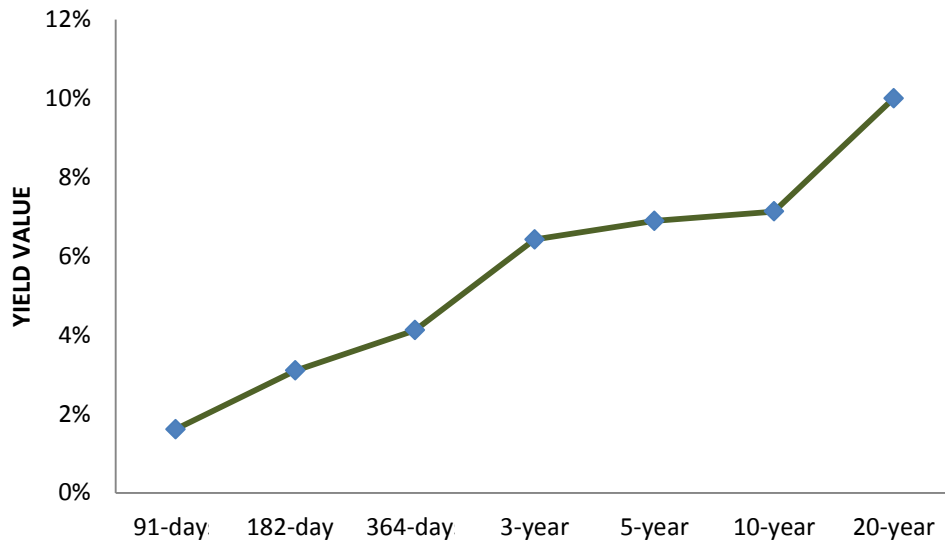
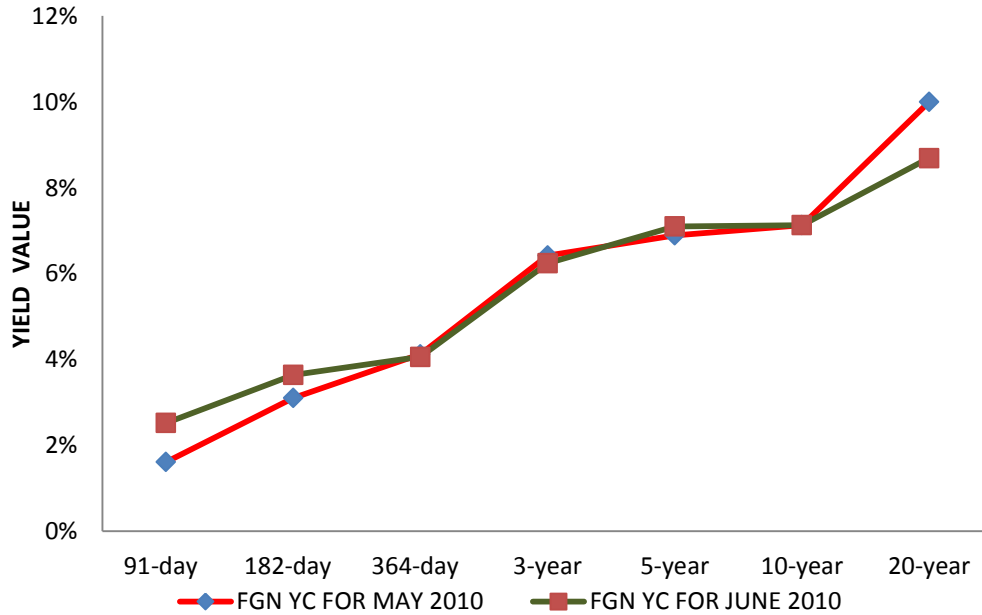


Figure 6: FGN Yield Spread for May/June 2010



CHAPTER 6: APPLYING CUBIC SPLINE MATHEMATICS TO SMOOTHEN NIGERIA'S ZERO-COUPON YIELD CURVE

In order to achieve a meaningful solution to the above equations, the cubic spline method imposes a number of conditions on each curve of the cubic equation. It is required that each cubic equation passes through its own pair of vertices.

Running equation (12) through vertices (0) and (1) (see Fig. 1) yields the following:

$$a_0n_0^3 + b_0n_0^2 + c_0n_0 + d_0 = \alpha \quad (15)$$

$$a_0n_1^3 + b_0n_1^2 + c_0n_1 + d_0 = \beta \quad (16)$$

In the same manner, running equation (13) through vertices (1) and (2) (see fig. 2) produces the following:

$$a_1n_1^3 + b_1n_1^2 + c_1n_1 + d_1 = \beta \quad (17)$$

$$a_1n_2^3 + b_1n_2^2 + c_1n_2 + d_1 = \gamma \quad (18)$$

Applying the same principle and running equation (14) through vertices (2) and (3) (see fig. 3) results in the following:

$$a_2n_2^3 + b_2n_2^2 + c_2n_2 + d_2 = \gamma \quad (19)$$

$$a_2n_3^3 + b_2n_3^2 + c_2n_3 + d_2 = \varphi \quad (20)$$

Essentially, the resulting yield curve, the coefficients of which are represented in equations (15 – 20) must be smooth and devoid of any "kinks" at the point where one cubic equation joins with the next.

The mathematical condition that ensures this firmly requires that the slope, as well as the convexity, of the adjacent equations be equal at the point where the pair of such adjacent equations converge, and thereby ensuring a smooth rollover from one equation to its adjacent pair.

Mathematically, this demands that the first and second derivatives of all adjacent equations be equal at the point where the pair of such adjacent equations converge.

At the point where the first cubic function meets the second cubic function (i.e. at vertices 0,1), which is captured in equations (16) and (17), and represented by:

$$a_0n_1^3 + b_0n_1^2 + c_0n_1 + d_0 = \beta$$

$$a_1n_1^3 + b_1n_1^2 + c_1n_1 + d_1 = \beta$$

the following first derivative conditions must be maintained:

$$3(a_0n_1)^2 + 2(b_0n_1) + 1 = 3(a_1n_1)^2 + 2(b_1n_1) + 1 \quad (21)$$

From equation (21) above, the following second derivative conditions must then hold true:

$$6a_0n_1 + 2 = 6a_1n_1 + 2 \quad (22)$$

Similarly, at the point where the second cubic function meets the third cubic function (i.e. at vertices (1, 2), which is captured in equations (18) and (19), and represented by:

$$a_1n_2^3 + b_1n_2^2 + c_1n_2 + d_1 = \gamma$$

$$a_2n_2^3 + b_2n_2^2 + c_2n_2 + d_2 = \gamma$$

the following first derivative conditions must be maintained:

$$3(a_1n_2)^2 + 2(b_1n_2) + 1 = 3(a_2n_2)^2 + 2(b_2n_2) + 1 \quad (23)$$

From equation (23), the following second derivative conditions shall equally hold true:

$$6a_1n_2 + 2 = 6a_2n_2 + 2 \quad (24)$$

It would be appropriate to impose a condition that the splines tail off flat at the end of the vertices (see equations (15) and equation (20). In formal terms, it is being implied that the second derivatives will assume zero values at their end points (see Figure 7). This implies that:

$$6a_0n_0 + 2 = 0 \quad (\text{Meaning the first spline assumes a flat beginning})$$

$$6a_2n_3 + 2 = 0 \quad (\text{Meaning the second spline assumes a flat end})$$

As indicated earlier, the above set of constraints provides a set of twelve equations, which if resolved can solve for the twelve unknown coefficients.

The solution to this set of equations requires a complex algebraic manipulation to which matrix algebra lends itself, as shown further below.

The set of twelve equations needed to find the twelve coefficients can be represented, using matrix algebra as:

$$[n]_j^i X [\theta_j] = [\vartheta\mu_{(j,t)}]$$

Given that $[\theta_j]$ could assume any value of the coefficient ($a_j, \dots, \text{to } d_j$), and $\vartheta\mu_{(j,t)}$ (i.e. the right hand numeric value of the equation) could assume ($\alpha, \beta, \gamma, \text{ or } \varphi$).

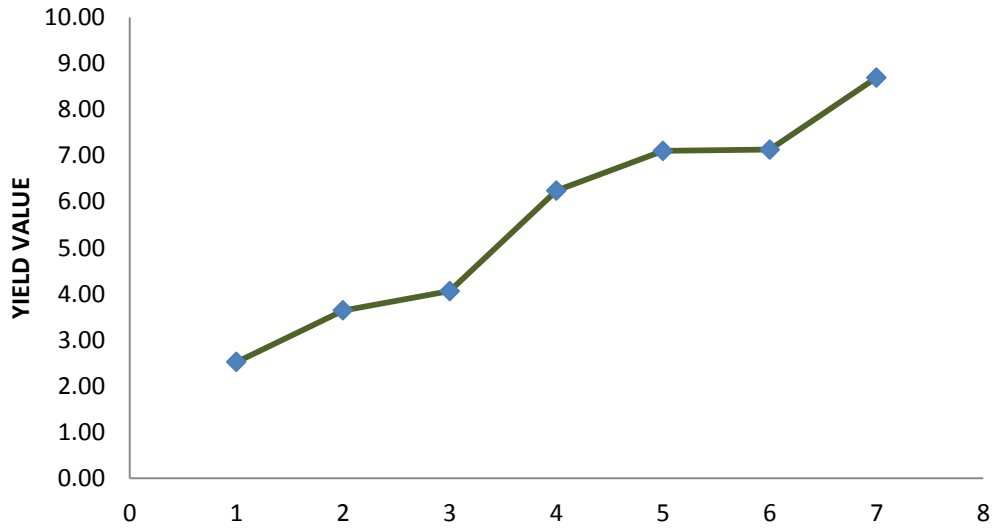
For the set of twelve equations indicated above, their matrix elements have been extracted, and are represented below:

$$\begin{bmatrix} n_0^3 & n_0^2 & n_0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ n_1^3 & n_1^2 & n_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & n_1^3 & n_1^2 & n_1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & n_2^3 & n_2^2 & n_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n_2^3 & n_2^2 & n_2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n_3^3 & n_3^2 & n_3 & 1 \\ 3(n_1)^2 & 2n_1 & 1 & 0 & -3n_1^2 & -2n_1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 6n_1 & 2 & 0 & 0 & -6n_1 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3n_2^2 & 2n_2 & 1 & 0 & -3n_2^2 & -2n_1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 6n_2 & 2 & 0 & 0 & -6n_2 & -2 & 0 & 0 \\ 6n_1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6n_3 & 2 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ a_2 \\ b_2 \\ c_2 \\ d_2 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \varphi \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

From here, the coefficients ($a_j, \dots, \text{to } d_j$) are found by inverting the $[n]$ matrix and multiplying its value by $[\vartheta\mu_{(j,t)}]$ (the right hand value of the equation) i.e.:

$$[\text{Coefficients } (a_j, \dots, \text{to } d_j)] = [n]^{-1} X [\vartheta\mu_{(j,t)}]$$

Figure 7: FGN Yield curve as at June 2010



CHAPTER 7 ANALYSIS AND INTERPRETATION OF NIGERIA'S ZERO-COUPON YIELD CURVE

A single yield curve that describes the cost of borrowing for all categories of obligor does not exist. Not in Nigeria, and certainly, nowhere else. Instead, there are yield curves describing the cost of borrowing for different groups of borrowers - sovereign or corporate - that reflects the cost of lending to an individual credit entity or obligor.

Nigeria's Zero-Coupon yield curve for June 2010 exhibits some remarkable features, and these shall be carefully espoused in our next discussion.

7.1 The Normal Yield Curve

Nigeria's June 2010 Zero-coupon yield curve is normal and positive, as shown in Figure 7. It reflects the notion that a return on an investment in Nigeria is anticipated to rise as the maturity of the investment lengthens. The positive slope of Nigeria's Zero-Coupon yield curve reflects market expectations that future real term returns on investment in Nigeria should adequately reward investors' risk. Figure 7 also demonstrates that investors are committed to keeping their investment locked-up in anticipation of future growth and economic development of the nation, and that a downward trend in economic activities is not anticipated, for now. A normal yield curve signifies that the future looks bright and the economy is guaranteed enough capital, *Ceteris Paribus*, to finance future growth and development. A more elaborate and standard version of a positive yield curve is presented in Figure 8. However, Appendix 2 shows that whilst the overall trend of a normal yield curve is generally expected to rise, a normal and positive sloping yield curve comes in more than one shape only.

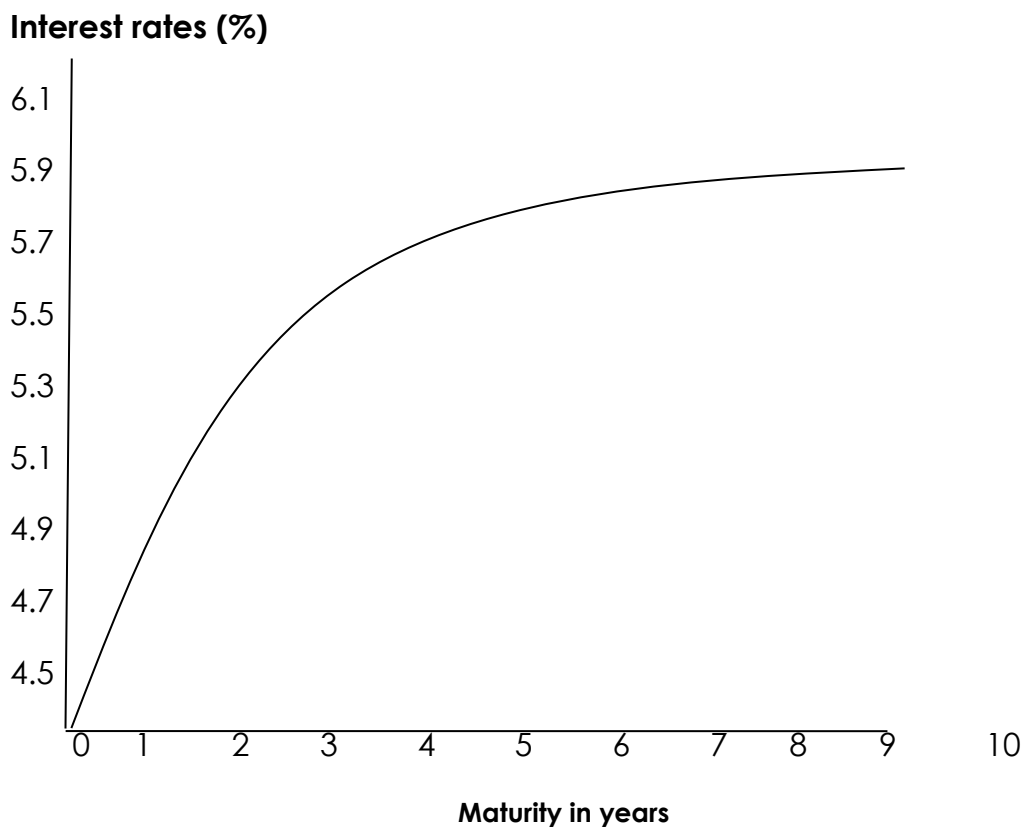
It must be noted that while a positive slope yield curve signals investors' readiness to make funding available for future economic development, it does not guarantee that they will not change their minds if better alternatives are presented to them elsewhere, or if anticipated adverse economic conditions make such a decision imprudent. Therefore, Nigeria should guide against excessive inflation, draconian taxation policies, irrational and wild volatility in interest rates, and exchange rates anomalies to maintain the present investment climate in her economy.

The country should also imbibe and nurture very prudent and sound fiscal and monetary policies at the macro level. In the same vein, political turmoil and all

other country-specific risk factors – internal political strife, corruption and disregard for the rule of law, which are likely to result in capital flight abroad should be avoided.

A positive yield curve could be analyzed in three different ways. According to Litterman and Scheinkman (1991), this could be done by analyzing the level, slope and curvature of the yield curve. Together, these three variables account for over 90% of changes in a Zero-Coupon yield curve, according to Martellini (2003).

Figure 8: Increasing or the Normal Yield Curve Interest rates (%)



7.1.1 The Level of Nigeria's Zero-Coupon Yield Curve

The level of the yield curve is equivalent to the numeric value of its yield at a given point-in time. Its value reflects a combination of real interest rates and

the average level of anticipated inflation at a particular moment. A generalized version of the yield level is depicted in Figure 8a.

From Figure 6, the level of Nigeria's Zero-Coupon yield curve for the 3-month debt instrument was approximately 2.5% for the month of June 2010. The expected level of interest was 1.8% in May 2010, giving a yield spread of approximately 4.5%. The spread was getting tighter as maturity lengthens to 6 months, and eventually disappears at year one. The interest rates remain almost unchanged for medium and long-dated instruments in Nigeria for the month of May and June 2010. The gap then widens significantly from year 10 onwards, and in line with the market expectation theory.

In general, the level of the yield curve, represents a perfect indicator of general level of interest rates. As such, a parallel shift in the level of the yield is a reflection of movements in the level of interest rates of all maturities by an almost identical margin or basis points (see Figure 8b). Where the shift is upwards, this could be interpreted as impending severe inflation. It could as well be an indication of a looming severe financial-economic shock at the macro level, reminiscent of the contagion effects of the recent 2007-2009 sub-prime global financial crisis. An upward shift in the level of the yield curve could also be as a result of an increase in the level of a nation's debt, especially where such debt is acquired at a high interest rate. Where the parallel shift is downwards, this could be an indication that a deflation or a contraction of economic activities is imminent. It could also be interpreted to suggest that severe capital flight or shortage is on the horizon. Nigeria did not experience this during the month of May and June 2010.

7.1.2 The Slope Oscillation of Nigeria's Zero-Coupon Yield Curve

Another way of analyzing a positive yield curve, such as that displayed by Nigeria's Zero-Coupon yield curve, is to take a look at its slope. This is a measure of the steepness of the yield curve. Martellini (2003) refers to this concept as the short-term or long-term spread of the yield curve. Figure 9a shows the effect of a scenario where the short-term interest rate is significantly higher than the long-term rate. This makes the overall yield curve to be less steep and results in a negative slope between times 'T' and 'T-1'. A different version of slope oscillation is offered in Figure 9a where proportionality in the relationship of the variables is assumed. A central bank's monetary actions exert a strong, but short-lived influence on the slope of the yield curve. According to Wu (2001) central bank's policy actions explain 80% to 90% of

the movement in the slope of the yield curve. The short segment of Nigeria's Zero-Coupon yield curve for May and June 2010 was responsive to the monetary policy interventions of the Central Bank of Nigeria, as its slope in Figures 4, 5 and 6 shows.

In essence, the slope of the yield curve provides a platform for monetary policy planners to measure the effectiveness of their policies by analyzing changes in the short-end slope of the yield curve as a result of monetary policy interventions.

It is, therefore, not impossible for the Monetary Policy Committee (MPC) of the Central Bank of Nigeria to perform useful sensitivity analysis on their monetary policy optional moves by calculating the effect of say, a 1% alteration in MPR on short-term interest rates. Doing this requires a deep knowledge of advanced differential calculus and stochastic calculus. Measured in basis points, the outcome of MPR alterations on interest rates could serve as powerful tool for fund and other asset managers operating on the interest rate arbitrage strategy. Likewise, a well-trained monetary policy analyst could use the same approach to guide and further enhance his/her monetary policy/decision-making scenarios by manipulating specific mathematical variables and interpreting the results accurately and precisely.

7.1.3 The Curvature of Nigeria's Zero-Coupon Yield Curve

This is a measure of variations in the shape (concavity or convexity) of the middle segment of the yield curve, and it further strengthens our understanding of the slope of the yield curve. Curvature differentials of a yield curve usually happen as a result of a temporary shock, or a drastic change in macro-economic or monetary policy. According to Willner (1996), the magnitude of change in the curvature of the yield curve is a reliable indicator of the impact or effect of alterations in monetary policy rates or levels of inflation. The curvature of the yield curve captures changes in the slope of the yield curve over a given time interval. As such, if well combined with the slope oscillation measure, the curvature of the yield curve can be a very powerful weapon for determining changes in the nominal level of interest rates. Such variations in the level of interest rates can then be decomposed to identify inflation-induced and real interest rate-induced changes. This can enhance inflation-management techniques, according to Mishkin (1990) and Estrella (2005b). In this regard, the yield curve curvature could serve as an effective monetary and inflation-targeting tool.

If this relationship is expressed mathematically, and the subject function rearranged, the curvature of the yield curve can be used to ascertain the magnitude of adjustment in MPR, or adjustments in the level of inflation required to achieve equilibrium in the money market. It can also be used to improve the approximation of inflation-targeting, as well as monetary-targeting to correct economic and/or financial market anomaly. The curvature of the yield curve is illustrated in Figure 10.

The curvature of Nigeria's Zero-Coupon yield curve for June 2010 exhibits a somewhat cyclical character (see Figures 6 and 7). It was concave at the short end of the duration (within one year duration), convex at between 1-3 year interval and again concave afterwards till around year 10. This is consistent with the fact that investors' long-term perception of Nigeria's economy, although optimistic, cannot be said to be even or constant. Rather, it looks cyclical and can change in either direction, depending on how well the market perceives the macro-economic policy of government. The future instability in oil prices is another factor that needs to be read into this as well, given that Nigeria is heavily reliant on petro-dollar, the price of which directly impacts on the economic wellbeing of the nation. Nigeria needs to work on comprehensively and consistently managing the volatility of her oil revenue.

Figure 9: The yield Curve Level Movement

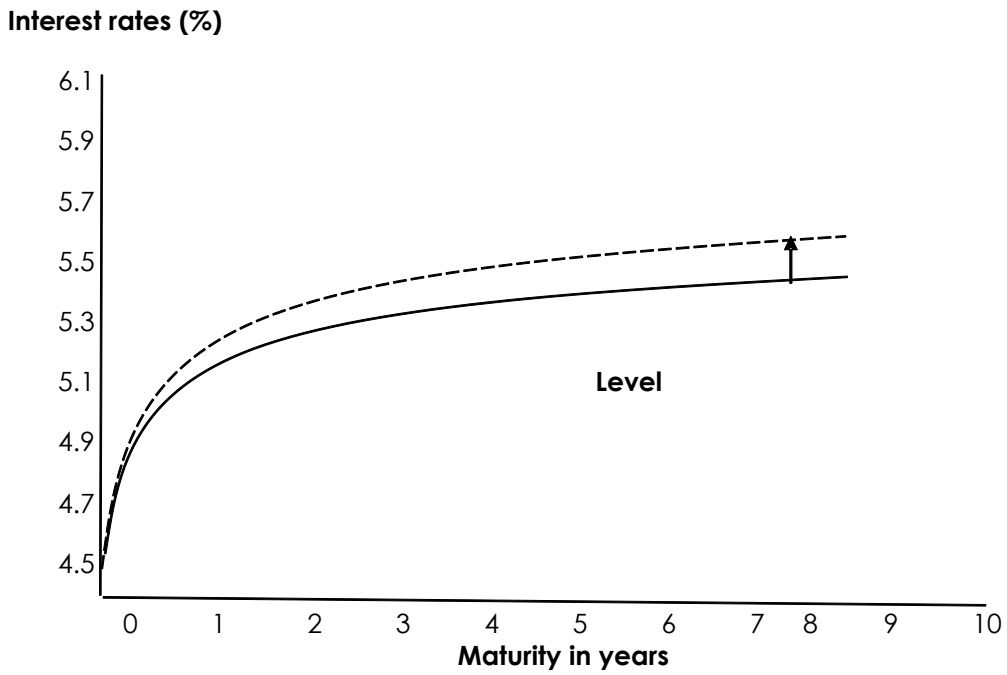


Figure 10: Upward and Downward Shift Movement in the Yield Curve

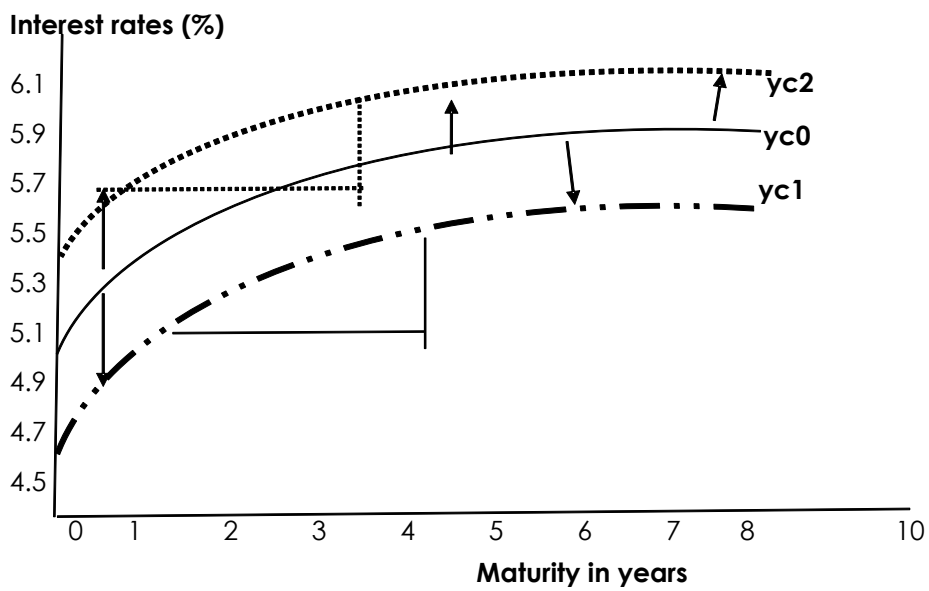


Figure 11: The Yield Curve Slope Movement (Non-Proportionality Assumed)

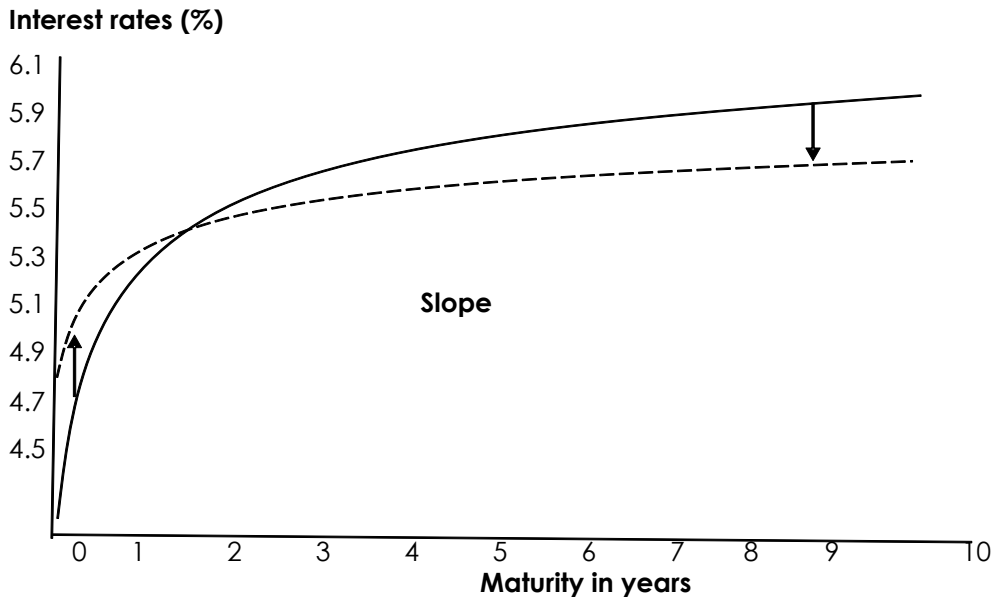


Figure 12: The Yield Curve Slope Movement with Proportionality Assumed

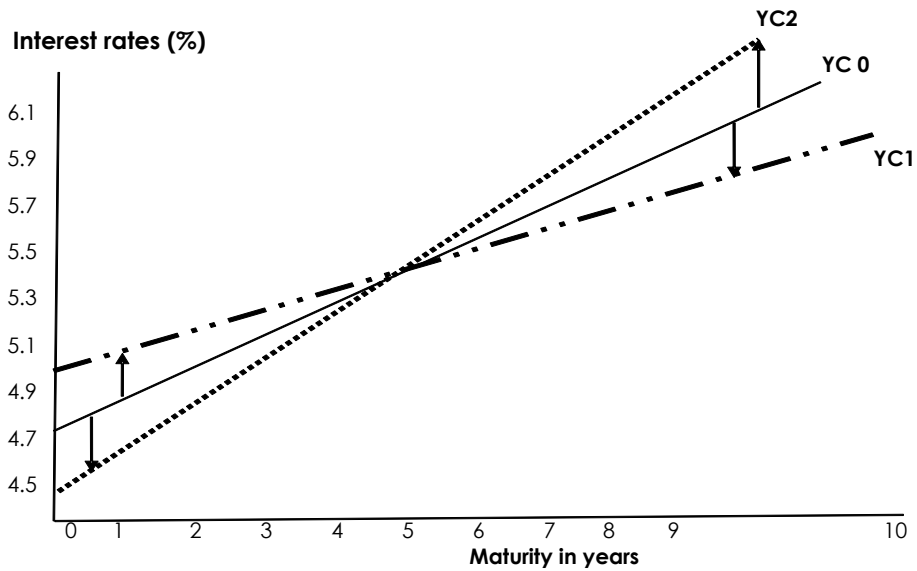
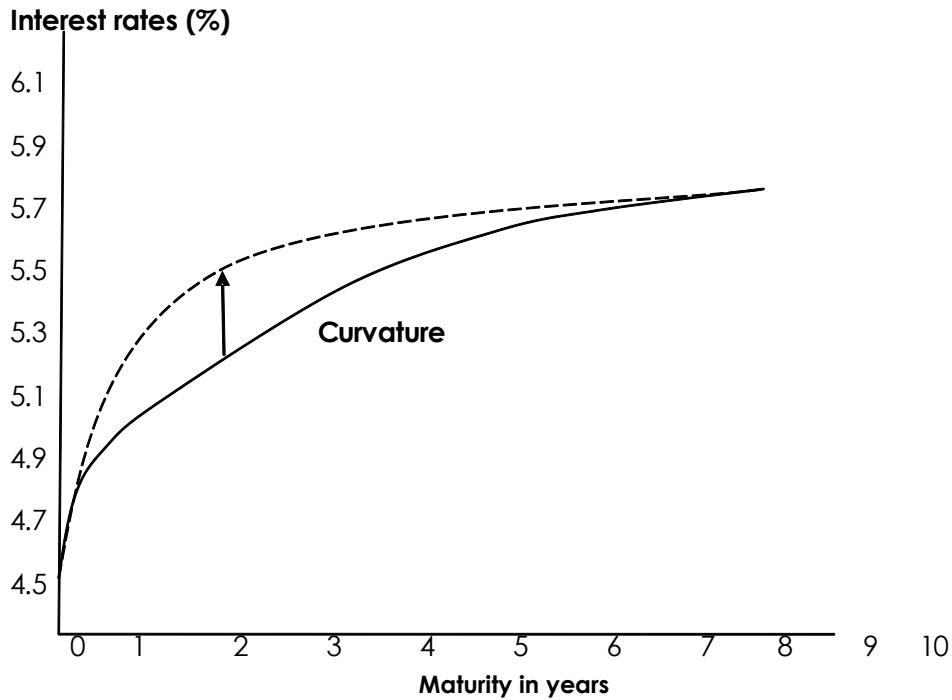


Figure 13: The Movement of the Yield Curve Curvature



7.2 The Inverted or Decreasing Yield Curve

An inverted yield curve is the direct opposite of a normal yield curve. It occurs when long-term yields fall below short-term yields. This is usually measured by subtracting 3-month treasury notes from 10 year treasury bonds. Where the outcome is negative, it indicates that short-term interest rates are higher than the long-term interest rates, according to Estrella (2005). It signals a very bleak outlook for the future as it reflects investors' preparedness to invest and earn less in return, in the short term, than earn even a lesser return in the future and be exposed to severe reinvestment risk on their investment portfolio. This mirrors investors' lack of confidence in the future economic outlook. It is an indication that there might be a shortage of funds for future economic development, and that the economy is likely to slow down. The severity of the expected economic slowdown is measured directly by the level and slope of the inverted yield curve.

A prudent monetary policy analyst would be guided by this scenario to plan the magnitude and timing of monetary injection –including buying of Treasury Bills from discount houses, defining a quantitative-easing target and/or fixing the minimum depository requirement, etc., to compensate for the expected liquidity shortage and maintain monetary equilibrium. The inverted or decreasing yield curve is illustrated in Figure 11.

Nigeria's Zero-Coupon yield curve neither exhibits this pattern nor shows its characteristics. Therefore, there is nothing to say, for now, in this regard about Nigeria.

7.3 The Flat and Humped Yield Curve

A flat yield curve manifests when all debt instruments of a particular credit entity are earning similar returns' regardless of their maturity dates. A flat yield curve is a signal of stagnation or uncertainty in the economy. It reflects indifference in investors' expected earnings at all time intervals. This is usually triggered by policy makers' indecision on very crucial economic policy, forcing investors to engage in a "wait-and-see" game with regard to future investment. It is a critical moment, which requires expert intervention and close monitoring. If handled improperly, it could easily tilt the economy towards a recession.

Empirical evidence shows that a flat yield curve is a rear occurrence, and it is more likely to be as a result of a somehow prolonged stationary interval at the point of inflection, where a downward-sloping yield curve is transforming into an upward-sloping yield curve or vice versa. An example of a flat yield curve is shown in Figure 12.

A humped yield curve, on the other hand, results from a skewed expectation of future interest rates. It could also manifest when short-term and long-term yields are equal and medium-term yields are higher than those of the short term and long term. Government debts are rolled over as they mature. The roll-over timing, the rates of interest on debts being rolled over, and the size of the debt being rolled over could result in certain segments of the yield curve exhibiting different slopes compared to others on the same curve. Such variations often manifest as bumps on the yield curve. A humped yield curve could, therefore, be a temporary manifestation of a replacement bond, or a roll-over of the existing ones on the yield curve.

Rampant roll-overs of Nigeria's FGN bonds in the past decade or so have been responsible for the bumps evident in Nigeria's Zero-Coupon yield curve as Figures 5, 6 and 7 demonstrate, and further supported by Figures 1, 2 and 3. The cyclical or bumpy nature of the Nigeria's Zero-Coupon yield curve, captured in June 2010 is explained. Examples of the humped yield curve are depicted in Figure 12a and Figure 12b.

The issue of a seemingly endless rolling over of Nigeria's existing FGN bonds has been of serious concern to experts, both within and outside the CBN. Experts are increasingly coming to the conclusion that Nigeria may no longer be able to sustain this policy.

Figure 14: The Inverted or Decreasing Yield Curve

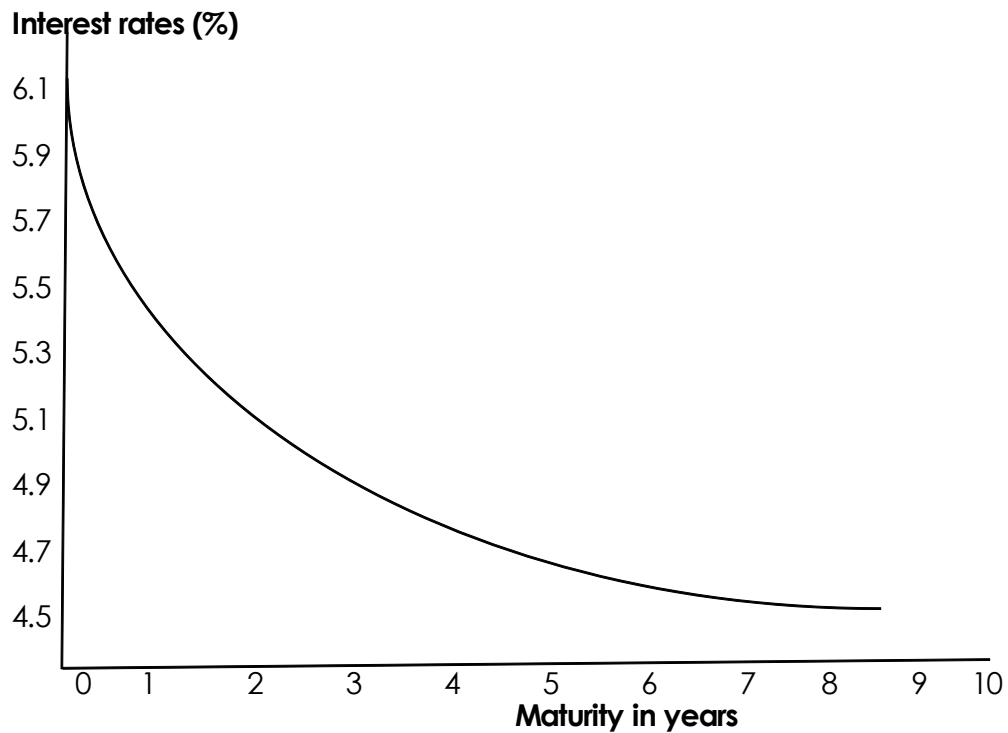


Figure 15: The Flat Yield Curve

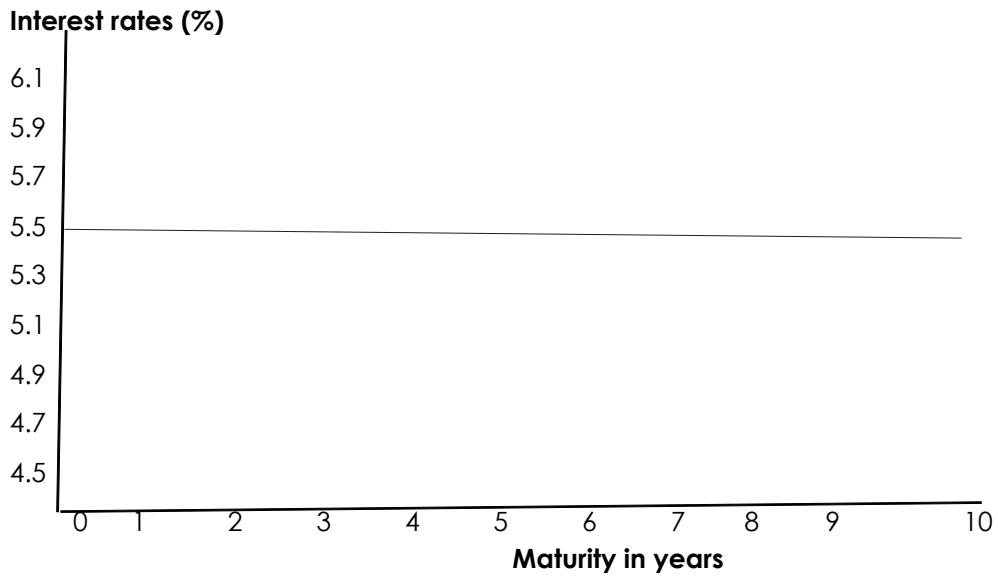
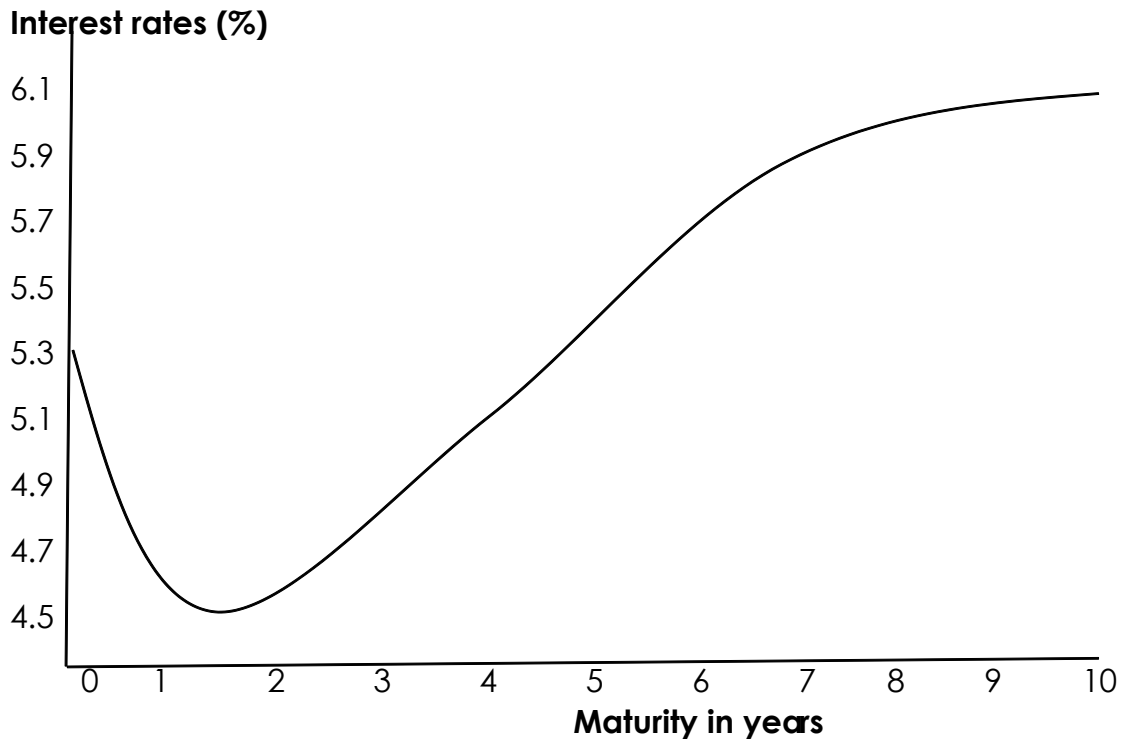


Figure 16: The Humped Yield Curve (Increasing on the Short End, and then Decreasing)



Figure 17: The Humped Yield Curve (Decreasing on the Short End, then Increasing)



PART IV

APPLICATIONS OF NIGERIA'S ZERO-COUPON YIELD CURVE

CHAPTER 8 THE USEFULNESS AND APPLICATIONS OF NIGERIA'S ZERO-COUPON YIELD CURVE

8.1 The Usefulness of Nigeria's Zero-Coupon Yield Curve

8.1.1 As a Tool of Fixed-income Analysis

The yield curve represents the market perception of the cost of a specific category of debt instrument at a specific time 'T' in the future, and it should be carefully analyzed and interpreted by experts. Its level, slope and curvature play a very crucial role in predicting the availability of credit (or lack of it) for future economic growth and development. The slope of the yield curve depicts the spread between long term (usually 10 year) and short term (usually three-month) interest rates. As such, the yield curve plays a very crucial role in predicting the prospects of future economic growth (where investors seem committed to providing future development credit) or recession (where the prospects of long-term development credit looks very dim).

8.1.2 As a Debt-instrument Pricing Tool

The yield provides invaluable input when it comes to pricing fixed-income financial instruments, such as corporate bonds, commercial papers, warrants, reverse warrants and convertibles. Its level serves as a benchmark, and provides a reliable guide, for pricing debt instruments of comparable qualities and characteristics. Taken from another view point, the yield curve represents the risk-free cost of debt from a national government to its various debt agencies - from discount houses to commercial banks - for onward extension to corporate entities and individuals. As such, the yield curve provides the benchmark against which all other yields are set, and other debt instruments are priced. In essence, a ten-year bond can be priced more accurately by using an appropriate and market-sensitive yield-curve-based discount rate for calculating the present value (market price) of such commodity instead of basing the price on the average discount rate over the length of its maturity years. Such approach is likely to exacerbate the reinvestment risk for such bond and to provide unreliable valuation of debt assets for arbitrageurs to feed on.

It must be emphasized that using the yield curve in pricing debt financial instruments applies more easily to plain vanilla or hard bullet bonds than it applies to option-embedded bonds, where further modifications are necessary. Appropriate pricing of debt instruments requires more than market-based yield; it is dependent on other quantitative inputs and variables (especially the day count basis, the frequency of coupon payment and the frequency of discounting) being computed properly and precisely.

The yield for other debt providers, other than that of a sovereign state, is determined by adding the credit spread (in basis point) to the official government yield. For example, a quote of (5% + 200 basis points) as the corporate yield for the Nigerian Breweries' credit request illustrates two clear elements. The 5% represents the debt-free government yield for a similar debt of identical characteristics and maturity, and the 200 basis points is the compensation accruable to the lender for taking the risk of extending credit to the corporate obligor or debtor (Nigerian Breweries), who cannot access government official rates directly. The basis point spread reflects the risk perception of Nigerian Breweries by the debt provider. The risk perception itself is determined by the credit score or rating of Nigerian Breweries and this is usually determined by credible and independent credit rating agencies, such as Moody's, Standard and Poor, Fitch, or Agosto in the case of Nigeria.

8.1.3 As a Credit Risk Quantification and Pricing Tool

The yield curve level could serve as the basis for ascertaining the appropriate risk premium or spread applicable in pricing municipal, agency or corporate bonds, or any other debt products. By default, it serves as the basis for fitting the yield curves for other non-governmental bonds.

As such, the yield curve facilitates the quantification and pricing of risk, especially where such a risk element has the yield curve level as its basis.

Technically, the monetary value of a given financial risk is equivalent to the additional basis points over and above the risk-free government yield curve basis. Using the decomposition method, the additional basis points and the monetary equivalent can, in turn, form the basis for quantifying other risk elements, including credit and interest rate risks, etc.

In view of the 2007-2009 global financial crises, the need to strengthen and enhance risk managing tools has been pushed onto the forefront of modern financial management. The yield curve remains unrivalled as a formidable and very reliable tool in this regard.

8.1.4 As a Re-investment Risk Management Tool

The yield curve can also be used to predict and manage re-investment risk - a unique and crucial aspect of fixed-income risk elements. This is made possible because the yield curve shows different levels of yield at different points in time.

As such, it is easy to identify the inflection point when a negative return in the yield curve sets in. It is at this point that the rate of yield growth at time 'T' starts to lag behind that at time 'T-1' (see Figure 9a and Figure 9b). This is tantamount to using the level, slope and curvature of the yield curve to estimate the severity, magnitude and duration of a negative trend (zone of re-investment risk) in the yield curve and price such risk accordingly. From all this, entire one can argue that the yield curve can serve as a reliable tool in the hands of a skillful bond strategist or a fixed-income analyst.

8.1.5 As a Relative Value Analytic Tool

It is technically feasible to analyze and compare yields across issuers, and thereby identify "cheap" or "expensive" debt products. This could be done in two different ways. One way, is to compare the yield on a debt instrument (e.g., a bond) of a given maturity against the corresponding market yield for the same bond.

Another way of performing relative value comparisons is to place two "identical" yield curves on top of each other and analyze any discernible outliers or anomaly. Provided the two categories of debt instruments being compared share common characteristics, both of the two methods of comparison enumerated above would identify undervalued or overvalued debt instruments, and characteristically invite the presence of "vultures" – the arbitrageurs. This phenomenon is demonstrated in Appendix 2.

Performing a relative value analysis is particularly valuable in pinpointing bonds trading above or below the market yield, and can show the size of such spread, if any. This would help fund managers to identify bonds to be disposed from, or added to, an investment portfolio.

For the hedge fund manager, the yield curve can provide a useful guide for spotting anomalies in the pricing of financial debt instruments, and in estimating the potential profit margin on offer. Knowing the magnitude of abnormal an profit margin available before taking a position can help the fund manager overcome a number of uncertainties, including performing a worst-case-scenario analysis and other forms of stress-testing without resorting to using complex alternatives, such as Monte-Carlo simulation or sensitivity analysis. As such, it can be argued that the yield curve simplifies an otherwise complex and rigorous analysis on fixed-income financial products or instruments.

8.1.6 As a Measure of Future Yield Levels, and an Indicator of Economic Recession

A well fitted yield curve exhibits a vast amount of highly valuable economic and financial information. Its shape illustrates market expectations of future interest rates. Consequently, bond traders and other debt market stakeholders analyze the present shape of the yield curve in order to understand market expectations of future interest rates, and direct their investments accordingly. Information provided by the yield curve is particularly useful to interest rates derivative traders and participants in "carry trade", as well as loan providers and bond investors, as in Watson (1991) and Estrella and Mishkin (1996). The level and slope of the yield curve, especially its 3-month - 10 year spread, are particularly useful in predicting economic recession, according to Estrella (2005a).

The usefulness of the yield curve in enhancing arbitrageur opportunities is illustrated by the ongoing post-crises financial turbulence in the Euro zone, with Ireland, Greece, Spain and, perhaps, Portugal presenting unique opportunities for carry traders.

Not only does the yield curve assist in predicting economic recession, it is equally capable of informing when such recession is most likely to commence, thereby allowing enough time for counter measures to be put in place.

Furthermore, the yield curve is capable of providing a very useful guide to an economy in recession regarding the magnitude of counter measures required to return an ailing economy back to the state of "status quo ante". This

particular quality of the yield curve has recently been tested in helping to assess the size of economic stimulus packages, and the quantitative easing measures required to stabilize the economies of the United States of America, the United Kingdom, Japan and other economies following the sub-prime mortgage crash of 2007-2009 and subsequent financial-liquidity crisis (see Sholarin 2009a and b).

8.1.7 As a Tool for Measuring and Comparing Returns on Various Debt Instruments Across Maturity Spectrum

Fixed-income analysts can use the yield curve to compare relative returns on debt instruments across a vast maturity spectrum. This is because the yield curve indicates expected investment returns that correspond to different maturity points in time. As such, the curve can help in quantifying and comparing investment returns on debt instruments of various debt providers and across a vast maturity spectrum. This could be of particular relevance to fixed-interest fund managers, fixed-income strategists, investment portfolio managers and other finance professionals. The curve assists in identifying the best returns on investment where debt instruments of similar characteristics are being compared.

8.1.8 As an Instrument for Pricing Interest Rate Derivatives

According to Martellini (2003) and Choudhry (2001, 2004), the price of derivative securities revolves around the yield curve.

Therefore, the yield curve represents an invaluable and very effective tool in pricing interest rate products, as well as their derivatives, including interest rate futures, forwards, options and swaptions. Derivatives are financial products whose value is dependent on (or derived on the basis of) the price of another underlying product. The yield curve shows the future pattern of prices of interest rates for various categories of debt issuers and, therefore, offers a reliable basis for pricing such underlying securities. The level, slope and curvature of the yield curve can provide glimpses of the future availability or scarcity of capital, as well as its magnitude. These indicators can as well be used in gauging the implied volatility of such future capital, as well as its size and timing. This is a major component in the Black-Scholes' option-pricing equation and other advanced derivatives pricing mathematics.

8.1.9 As an Effective Tool for Liability-driven Financial Asset Management

The yield curve can be applied to improve asset-liability management by helping to ascertain the size and duration of the investment assets required to match the expected future liability of financial institutions. This could have special application in pension fund management/and in complex actuarial arrangements. A mismatch in the asset-liability ratio could then be used to adjust the investment pattern or to renegotiate liability. According to (Elliot, 2004), the government yield curve has the potential of being used to improve the management of a "pension hole" or a related asset-liability disequilibrium, and help determine the magnitude of return required to match a given level of liability.

8.1.10 As a Liquidity Management and Credit Rating Tool

The yield curve is an expression of the market's perception of future availability of credit or lack of it. The movements in its level and ratio can, therefore, serve as useful guide for rating agencies in ascertaining their future credit availability or its absence. Such movements can then be used in gauging refinancing risk, Credit Default Swap or similar debt-based financial instruments. It can also be used in monitoring liquidity, or lack of it, in a financial market.

The level of the yield curve is also capable of guiding a monetary policy analyst in managing liquidity by determining the magnitude and timing of monetary withdrawal (to mop-up) or monetary injection (to top-up) in order to ensure equilibrium and achieve stability in a financial market.

8.2 Applications of the Yield Curve in Financial and Monetary Policy Enhancement

The yield curve represents a unique and highly valuable tool for technical analyses. Yield curve users can be separated into two distinct categories as follows: financial market practitioners and financial market regulators. This research paper investigates the relevance and usefulness of the yield curve to financial market regulators, which include central banks. Thus, specific attention shall be paid to how the Monetary Policy Committee of the Central Bank of Nigeria, in particular, can most benefit from the yield curve as an analytic tool for enhancing monetary policy management in Nigeria.

Central banks can use the yield curve to determine the implications of movements in market interest rates. Central banks and government treasury

departments can also analyze the yield curve for its unique information content, firstly for ascertaining forward interest rates and, secondly, for estimating permissible, as well as sustainable inflation levels. Available data shows that the negative net return on investment in Nigeria was particularly high (sometimes totaling (-2.5%)) between 2000 and 2010 as a result of a mismatch in nominal interest rates and the market's perception of the level of inflation.

The yield curve can be used to capture the effectiveness of monetary policy management and its various tools. Every alteration in monetary policy tools (be it an adjustment in quantitative easing, MPR, Minimum Depository Requirement, or in aggregate money supply) is directly reflected on the yield curve in the form of, altering its short-segment level, slope or curvature. Consequently, a well-trained monetary policy analyst can use the yield curve to capture, quantify and analyze the effects of such monetary policy tools as well as measure their relative effectiveness. After correcting for time lag effect, the policy maker can use the yield curve further to guide his judgment with regard to the appropriateness of an expansionary or contractionary policy, as well as the timing of such monetary policy intervention. In this regard, the yield curve can play a crucial role in enhancing and facilitating monetary policy management (Waggoner, 1997).

Research has shown that a strong and positive correlation exists between the slope of a yield curve and monetary policy shocks or interventions. According to Wu (2001), 80% to 90% of movements in the slope of a yield curve can be attributed to, or explained by, monetary policy shocks and interventions. The empirical investigation performed by Wu, (2001) further shows that such influence is temporary, and would dissipate in less than three months. A similar relationship was found to exist between the level of inflation and the government yield curve, according Ang and Piazzes (2001).

Furthermore, central banks and government treasury departments may find the slope and level of the yield curve particularly useful in determining forward interest rates, and also for inflation-targeting, according to Estrella and Mishkin (1995).

With regard to inflation-targeting, an upward or downward shift in the yield curve could signal the effect of inflation or deflation respectively. If well decomposed, it is possible to separate the shifts attributable to increases in

real interest rates, as well as the shifts attributable to changes in the CPI or inflation level, according to Ameida et al. (1998) and Ang and Piazzes (2001). The information so gathered can then be used to manage inflation-targeting exercise, and thereby enhance monetary policy management of a nation's economy.

Using the yield curve to manage forward interest rates is equally feasible. The forward interest rate, at a given time in future, is already a feature of a standard yield curve. However, short term, temporary shifts in the yield curve that usually last for few days or weeks, should be ignored and disregarded as they simply constitute "flukes" or market spasm; they are not representative of any trend or pattern. An independent investigation of Canadian financial markets by Wu (2001) corroborates this assertion. This issue will be revisited later in this research report.

8.2.1 The Yield Curve and Inflation-targeting.

The yield curve is capable of serving a central bank in its monetary policy management function, especially in inflation-targeting. It is a very useful and reliable tool in predicting the present, as well as the future levels of inflation. The level of the yield curve indicates the nominal level of interest rate which, if decomposed, can show the level of real interest rate, as well as the level of inflationary spread. Decomposition can be performed with advanced quantitative tools, such as factor analysis or advanced econometric models, as in Watson (1991) and Ameida (1998).

In essence, it is possible for the monetary policy of a central bank to be guided by the yield curve in defining and targeting the permissible level of inflation that the economy can tolerate without disturbing the national macro-economic equilibrium Mishkin (1990) and Estrella (2005b).

The work of Dr. Michael Olufemi Ojo (an erudite research scholar at the Central Bank of Nigeria) deals extensively with the issue of inflation-targeting with specific applications to Nigeria.

8.2.2 The Yield Curve and Monetary-targeting

Another area of usefulness of the yield curve lies in its capacity to enhance monetary policy management by providing a very useful guideline in defining the monetary aggregates an economy needs to remain in equilibrium and achieve price stability. The desired value, in its aggregate form, is equivalent

to the level of the yield curve minus the permissible level of inflation. This value can rise during an inverted yield curve season to compensate for shortage of funds, and vice versa.

A very challenging caveat must, however, be introduced here. The truly global nature of financial markets means that it is no longer possible for a country to single-handedly determine the aggregate money supply to or from a well-integrated economy operating within the global financial market.

For countries on course towards global financial integration, including Nigeria, monetary-targeting as a tool of monetary policy management will soon outlive its usefulness and will be abandoned in favour of self-regulated movements of funds, according to the forces of the global financial markets. It will cease to be a feasible tool of monetary policy management as soon as countries become fully integrated into the global financial system.

8.3. The Yield Curve and Oil Price Volatility

The level and slope of the yield curve can also serve as a gauge of oil price volatility. A preliminary investigation of the effect of the price of crude oil on Nigeria's Zero-Coupon yield curve shows that there is a negative correlation between the two variables. As the price of the crude oil falls, the demand for additional funding by government increases. This drives up the cost of borrowing to service and roll over the existing debts of government, and the yield on government bonds rises. This then cascades down to other financial intermediaries who rely on government, through discount houses, as their primary source of loan capital. It is, therefore, possible to quantify the effect of oil price volatility (for example, of Nigeria) by measuring changes in the slope and level of the Nigeria's Zero-Coupon yield curve. However, as there are several other factors likely to influence the slope and yield of the yield curve other than the price of crude oil, it will be appropriate to hold all other factors constant – *ceteris paribus* – whilst such a relationship is being investigated. An alternative approach would be to perform a multiple regression analysis and isolate the influence of each individual variable.

CHAPTER 9 THE IMPLIED FORWARD YIELD CURVE FOR NIGERIA AND ITS APPLICATION

Future financial transactions are highly relevant in modern financial markets. For a premium, forward contracts enable counterparties to a financial transaction to agree a pre-determined strike price for products to be delivered at a future date. With regard to the time structure of interest rates, forward rates can be derived from spot interest rates, and are referred to as 'implied forward rates'. Forward rates are of particular importance in national economic management.

Their forward-looking characteristic enables them to play unique roles when it comes to ascertaining the Monetary Policy Rate and, by proxy, estimating the inflation targeting range.

It must be emphasized that, although the use of the yield curve is well documented and well understood as an instrument for monetary policy enhancement, the introduction of the forward yield curve (mainly by the US Federal Reserve and the Bank of England) is relatively new and still evolving.

The difference between the forward rates and the spot rates is referred to as the 'term premium' whose unique set of characteristics are of particular usefulness to financial regulators, especially those vested with the task of monetary policy management. The term premium provides valuable information and indications about expected changes in spot rates of interest and also, most importantly, the magnitude of such changes.

Because the forward yield curve is a plot of the forward rate against the term maturity value, it represents the spot bond yield as at the forward date. Effectively, it is the yield of a zero coupon bond that is purchased for settlement at the forward date. As the forward rate is derived at the present moment, using data from a present-day yield curve, it would, therefore, be technically wrong to construe it to be a forecast of the spot rate as at the forward date.

Regardless of this assertion, however, the forward rate value plays a very crucial role in financial markets. The forward rate is used by market makers to quote the daily prices for dealing in financial transactions designed for execution at future dates. The forward rate represents the best expectation of

future interest rates (given all that is known in the market up till the very last moment of calibrating the rate).

In this regard, the implied forward rate can, therefore, be interpreted as reflecting the market's expectation of future short-term interest rates. Consequently, forward rates provide reliable information regarding the future level and direction of interest rates.

Forward yield curves are better suited to monetary policy analysis than the spot yield curve because of the extra and advanced features it offers over and above the zero-coupon yield curve. Furthermore, the superiority of forward rates is entrenched in the fact that, it can be split into short-term forward rates.

This makes it possible to decompose forward rates into short-term and long-term segments in a more straightforward fashion than the spot rate.

It must, however, be noted that the level of mathematics required for a full understanding of even intermediate concepts in finance, such as this one, is very high, according Choudry (2004), and cannot be exhaustively addressed in a one-off paper of this nature.

The zero-coupon yield curve is most ideal for deriving the implied forward yield curve, provided that such zero-coupon bond is derived on the basis of coupon-based bond prices, such as the one used in generating Nigeria's Zero-coupon yield curve. In this instance, the forward rate applicable to a bond is the spot yield on the bond as at the forward date. In other words, this is the yield of a zero-coupon bond that is purchased for settlement at the forward date.

Given a set of pure discount bond prices $b(t, T)$, it is possible to calculate a set of forward rates defined on date t as $f_t(T_1, T_2)$. This forward rate can be locked in on date t for transactions starting on date $T_1 \geq t$ and maturing at $T_2 \geq T_1$. for, $T_2 \geq T_1 \geq t$.

The procedure for the application of the forward yield can be illustrated, using two investment strategies. In strategy one, an amount (in Naira) invested on date t at a rate of y_1 earns a return of $(1 + y_1)^{T_1-t}$ on date T_1 . If the return is

then sold forward at a forward rate of $f_t(T_1, T_2)$ on date T_1 , the return on date T_2 will then be $(1 + y_1)^{T_1-t} \times [1 + f_t(T_1, T_2)]^{T_2-T_1}$.

Please, note that $f_t(T_1, T_2)$ connotes the forward rate at date t for the period starting date T_1 , and ending date T_2 .

Since the forward rates are established on date t , then such a transaction is adjudged to be risk-free and, any credit risk which might arise due to non-performance of any of the counterparties is, for the purpose of analysis, ignored.

In the parlance of structured finance, an equally valid investment strategy, which yields the same amount of return as on date T_2 above, and using strategy two, will be to simply invest the same amount on date t at a rate of y_2 to earn a return of $(1 + y_2)^{T_2-t}$ on date T_2 .

In order to eliminate arbitrageur's opportunity, the two investment strategies above must satisfy the following condition of equality:

$$(1 + y_1)^{T_1-t} \times [1 + f_t(T_1, T_2)]^{T_2-T_1} = (1 + y_2)^{T_2-t} \quad (25)$$

From the analysis above and arguments, the following assertion holds true:

A term structure of interest rate contains an appropriate element and relevant information which is useful for calibrating the forward rate of interest, and such forward interest rate can be extracted from the spot (i.e. the zero-coupon) term structure, using synthetic extrapolation methods, such as those offered in equation (25).

In principle, forward rates can be computed for various forward dates in the future, using equation (25) above. However, a number of limitations must be borne in mind when using this concept in practice. The first limitation arises from the fact that there are insufficient benchmark maturities in Nigeria's term structure spectrum. As at the moment of preparing this report, Nigeria has eight benchmark maturities in her sovereign bond universe: 3 months, 6 months, 1 year, 3 years, 5 years, 7 years, 10 years and 20 years (the 7-year duration is ignored for lack of data). This means that there are significant gaps in the 5-year to 10-year maturity segments, and in the 10-year to 20-year maturity segments. Another hindrance is the fact that the frequency of FGN

bond auctions in the 20-year segment is very low in Nigeria, thereby creating an unusually high number of empty cells in its bootstrapping. This limitation indicates that a sizeable number of on-the-run issues may not trade at or near their par value on dates further away from the auction date.

Notwithstanding these limitations, this paper demonstrates that it is possible to calibrate and fit a good spot rate (zero-coupon) curve for Nigeria, based on limited information, and to calibrate and fit the implied forward yield curve as required.

The yield value calibrated for Nigeria is consistent with the fundamentals of market expectations and risk premium theories. It also upholds the general notion that an investor or a bond trader must be indifferent in choosing to invest on date t at a rate of y_1 earning a return of $(1 + y_1)^{T_1-t}$ on date T_1 , and with the right to roll his portfolio over to T_2 at a forward rate of $f_t(T_1, T_2)$ on date T_1 , in order to earn $(1 + y_1)^{T_1-t} \times [1 + f_t(T_1, T_2)]^{T_2-T_1}$ as proceeds on date T_2 .

As the focal point of this paper is to devote a substantial amount of attention to monetary policy enhancement for the Central Bank of Nigeria, it is the 3-months to 12-months segment of the forward FGN yield curve (the major tool of monetary policy analysis) that will be relevant for this investigation. For this purpose, the forward yield of duration extending beyond one year shall be of no relevance.

The FGN's combined spot and forward yield table is offered in Table 3, and the yields are contained in Figure 13. Its analysis is offered in Figure 14.

Table 3: FGN Spot and Forward Yield

ISSUE DATE	TENOR	COUPON YIELD%	ZERO-COUPON YIELD%	FORWARD RATE%
17-June, 2010	91-Day	2.50	2.52	2.52
17-June, 2010	182-Fay	3.51	3.64	4.77
10-June, 2010	364-Day	3.90	4.06	4.53
25-June, 2010	3-Year	5.50	6.24	6.58
25-June, 2010	5-Year	4.00	7.10	7.35
23-June, 2010	10-Year	7.00	7.13	7.25
25-June, 2010	20-Year	8.50	8.69	8.77

Please note that in applying equation (25) to arrive at the above forward rate figures, 91-day duration is treated as one quarter, 182 days duration as two quarters and 20-year duration is treated as eighty quarters, and the exponents are adjusted accordingly. The mathematics behind these forward rate figures are contained in the appendix (1).

Figure 18: The Zero-Coupon vs. the Forward Yield Curve for FGN June 2010 Bond Series

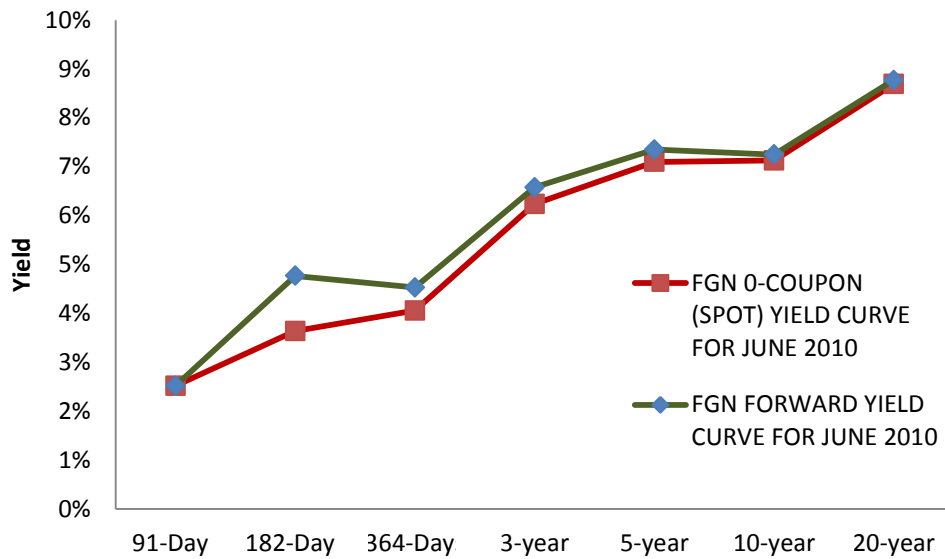


Figure 19: FGN Zero-Coupon vs. Forward Yield Curve for June 2010

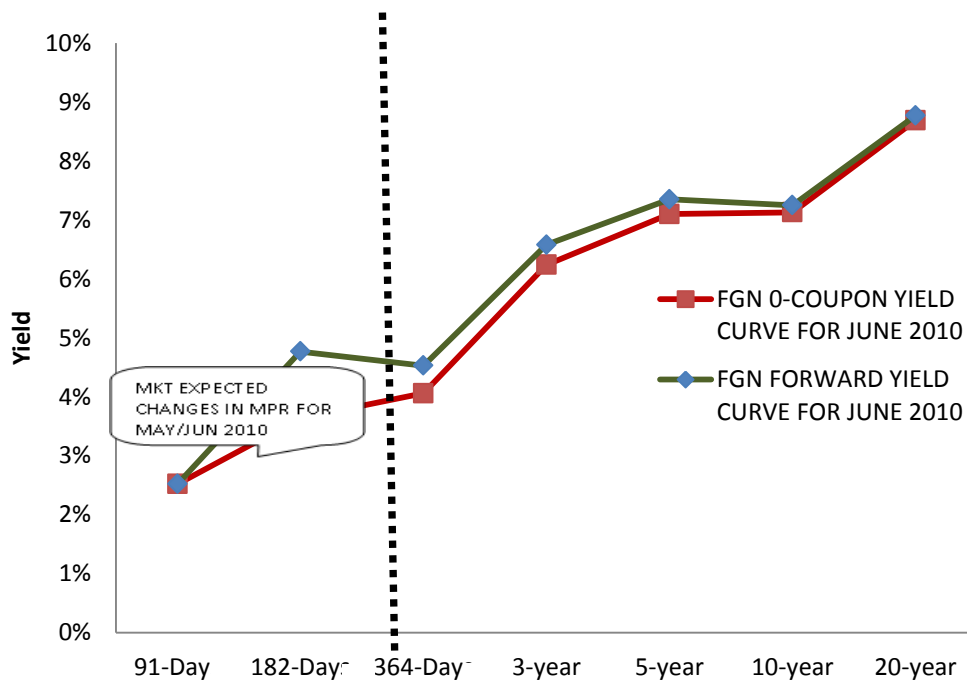


Figure 14 illustrates a forward-looking, stress-testing of Nigeria's MPR, using the forward and zero-Coupon yield curves to determine the optimal range of MPR alteration available to the Monetary Policy Committee (MPC) of the Central Bank of Nigeria between June and September 2010. This range, which is equivalent to $[1 + f_t(T_1, T_2)]^{T_2 - T_1}$ (where T_1 is 91 days, T_2 is 182 days and f_t is the forward interest rate as at the initial period of T_1), is adjudged to be 110 basis points, according to Figures 13 and 14.

According to Campbell (1997), the forward rate (otherwise known as the marginal rate of return) is equal to the spot rate of return, plus the rate of increase of the spot rate, multiplied by the sum of the increases between t and T . For this reason, if the spot rate is flat or constant, then the forward rate would be equal to it. Market experience has shown that an increasing spot rate does not necessarily translate into an equally increasing forward curve. It is possible for the correlation coefficient between a spot rate and a forward rate to assume a positive or a negative relationship. This is not uncommon during periods of economic uncertainty or adverse market expectations.

The relevance of using the forward yield curve in this manner is that it guides the decision of the MPC members in choosing a new MPR, which takes account of the inflation level or target and, as such, it effectively guides against choosing an MPR capable of implying a negative real interest rate.

The shape obtained for Nigeria's zero-coupon and forward yield curves (see Figure 13 above) is consistent with the established notion that the forward yield curve must follow the general pattern of its zero-coupon foundation, and that it must also lie above it. Nigeria's forward yield curve lies above its Zero-Coupon yield curve, and this lends credibility to the assertion shown in Figure 13. The assertion is further corroborated in Figures 15 and 16. This assertion is consistent with the findings of established authors, including Choudhry (2004), Martellini et al. (2004) and Fabozzi (2006).

The implied forward yield curve is particularly useful for determining the relative value of bond instruments, for the pricing of new bond instruments when trading in a financial market, regardless of their coupon value.

Figure 20: Monetary Policy Rate Forecasting using Ordinary Zero-Coupon yield
Differential (In Retrospective Form)

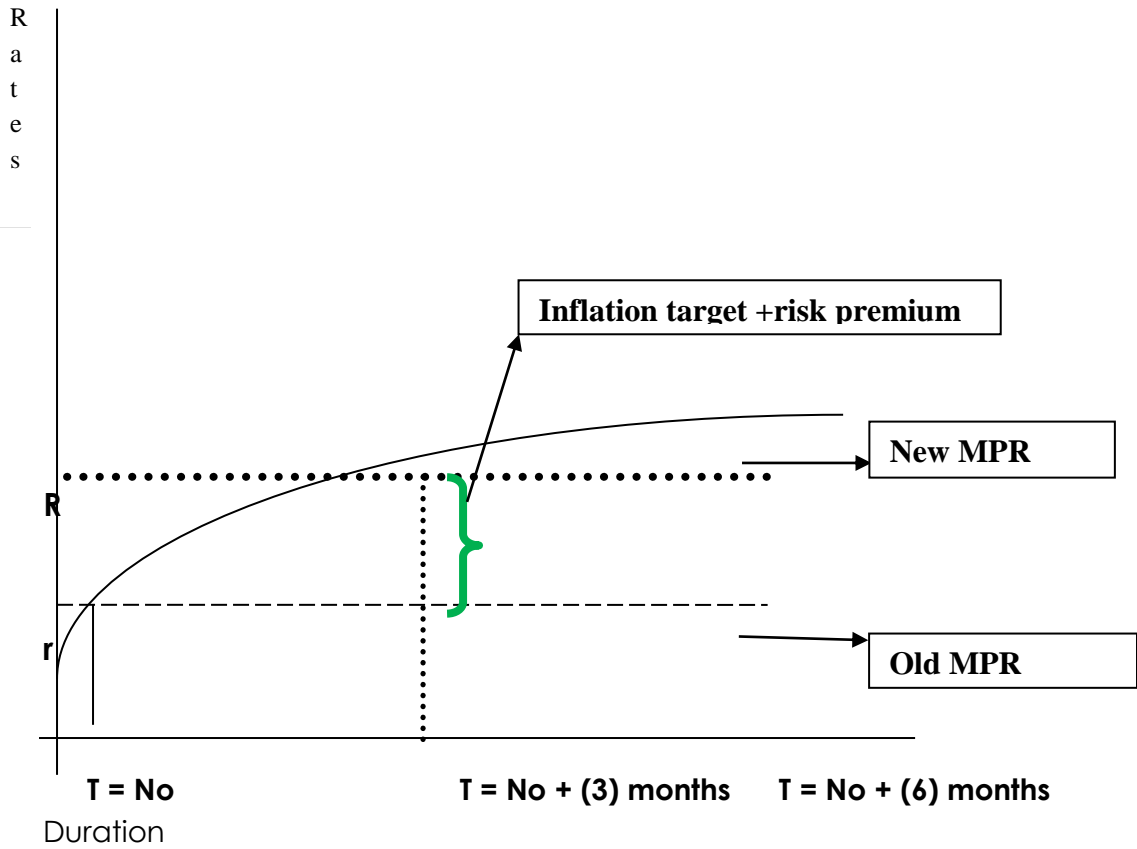
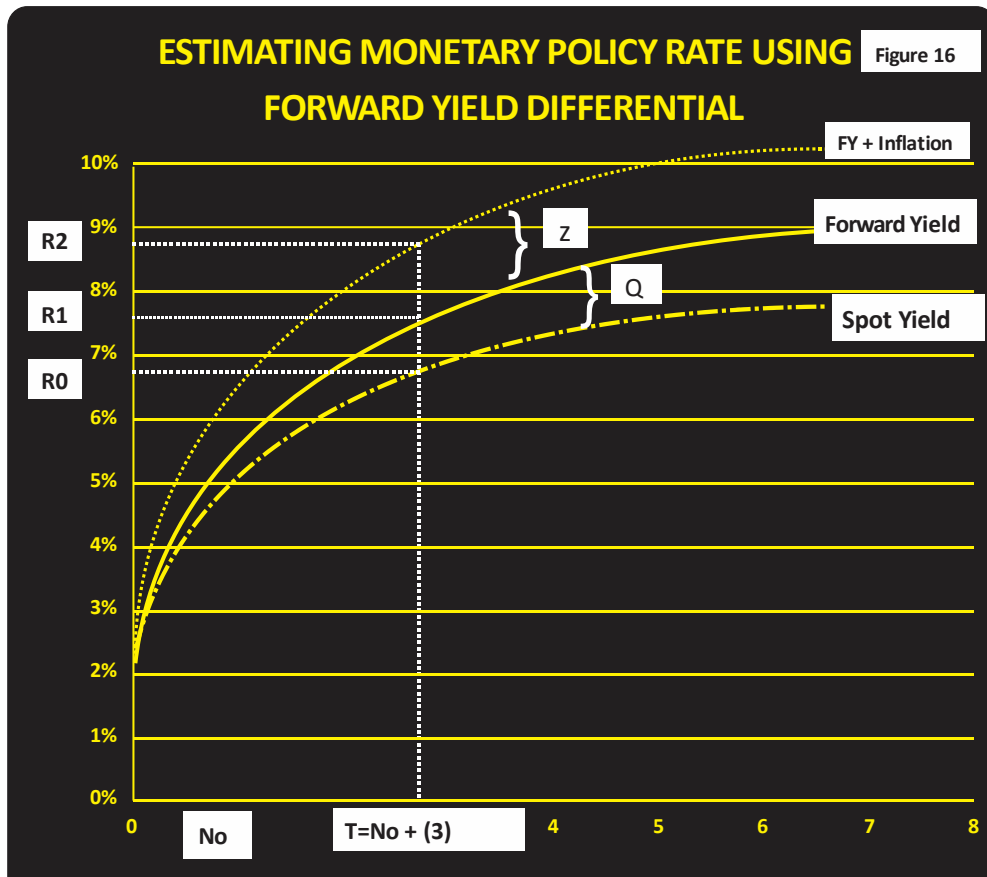


Figure 21: Estimating Monetary Policy Rate using Forward Yield Differential



Where:

No = 3 months

R0 = Spot yield at time $T = n$, and equivalent to (MPR) or base rate at time $T = n$

R1 = Expected base rate, and (MPR) equivalent at time $T = n + 3$ (in real value)

R2 = Expected base rate, and (MPR) equivalent at time $T = n + 3$ (in nominal value)

Z = Inflation target

Q = Spot-Forward yield differential

CHAPTER 10 A NOTE ON HOW THE PRICE, YIELD AND COUPON RATE OF A BOND ARE INTERRELATED

When it comes to analysing fixed income, the major inputs are the coupon rate, the required yield, and the price. The theoretical fundamental of any fixed coupon instrument is such that the relationship among the coupon rate, the required yield, and the price of a bond follows the following relationships: *When the coupon rate equals the required yield, then the price of the instrument equals its par value.*

This implies that:

When the price of a fixed income instrument equals its par value, then the coupon rate equals the required yield.

As the coupons on a bond (in this case, straight bullet bonds) remain fixed throughout its duration time, while the market yields often fluctuate to reflect market conditions, the adjustment for any distortion in the coupon-yield relationship is governed by the notion that:

When the coupon rate of a bond is less than its required yield, then the price of such bond instrument is less than its par value.

It follows from this that:

When the price of a bond instrument is less than its par value, then the coupon rate is less than the required yield.

When the coupon rate of a bond is higher than the required yield, then its price is higher than its par value.

Conversely:

When the price of a bond is higher than its par value, then its coupon rate is higher than its required yield.

SUMMARY AND CONCLUDING REMARKS

This paper presents, in four parts of the ten (10) chapters, the report of a research project commissioned by the Central Bank of Nigeria to calibrate, fit and analyze Nigeria's Zero-Coupon Yield Curve. The paper offers a theoretical background of the yield curve, and analyses areas of possible application of the yield curve. Specific areas of application of the yield curve in monetary policy management - including inflation-targeting - are given considerable attention. These scenarios formed the basis for justifying the usefulness and relevance of the yield curve as a tool for strengthening and enhancing financial economic policy and management. The benefits of the yield curve in pricing debt instrument, for the analysis of risk, and the pricing of interest rate derivatives are also offered and discussed.

The yield curve comes in various shapes and is peculiar to the individual debt issuer or obligor. While the rising or up-ward sloping yield curve indicates a stable and prosperous future economic prospect, the inverted yield curve suggests the opposite, indicating the strong likelihood of an economic slow-down or recession. Somewhere in the middle of the two extremes is the humped yield curve, characterized by a sharp and opposite turn (upward or downward) in the direction of the yield curve immediately after the short-term segment. Added to these shapes is the yield curve with a constant yield, regardless of the maturity of the debt obligation. This is referred to as the 'flat yield curve'. The normal yield curve is the most common of all these possible scenarios.

In analyzing the shape of Nigeria's Zero-Coupon yield curve, special attention is given to analyzing its level, slope and curvature. These indicators contain very useful and reliable financial and economic information, which have been amply articulated and extensively discussed in the research paper.

The information or output obtained from the yield curve can serve as the yardstick with which to gauge the authenticity of similar economic indicators acquired by alternative means.

The benefits of using the yield curve as a tool of economic analysis are enormous. Such benefits certainly outweigh the costs (both direct and indirect) of calibrating and fitting the curve.

Compared to the yield curves of such countries as the United Kingdom, Germany and South Africa, Nigeria's yield curve exhibits more grid points at the short end of her term structure, with progressively fewer points as the maturity lengthens. This suggests that far more short-dated bonds are traded in Nigeria than long-dated ones, an indication of under-development of her pension-related or long-term infrastructure development (including energy) bonds. The nature of Nigeria's data sets makes the fitting of Nigeria's yield curve particularly challenging.

The short segment of the FGN bond (mainly populated by TB) is the place where any anticipated changes in the base rate (MPR) would be manifested.

RECOMMENDATIONS

The following measures are recommended to the Central Bank of Nigeria with regard to being self-reliant in calibrating and fitting future zero-coupon yield curves for Nigeria.

- Set up a Team of dedicated staff to gather and process YC input data for Nigeria on a daily basis and as shown in Appendix 3;
- From amongst the Team so set up, carve out a sub-team to calibrate Nigeria's yields, fit its curves, and offer consistently sound and robust analyses of the yield curves on a daily basis;
- Through adequate training and capacity building, empower the Team to synthesize, analyze and enhance CBN's economic and financial policy formulation, as well as its supervisory function in the fixed-income segment of Nigeria's capital market;
- Continually update CBN's knowledge base, and remain several steps ahead of market practitioners in its forecasting capacity;
- Enhance the corporate end of Nigeria's fixed-income market by providing the necessary regulatory, institutional and infrastructure capacity to re-invent the corporate bonds in Nigeria;
- Extend this endeavour to include interest rate modelling for Nigeria;
- Allow for financial derivatives (swap, forward, options and swaptions), including CDS, securitization, CDOs CDO of CDOs, sound credit-rating institutions; and
- Embark on aggressive road shows to showcase the renaissance of Nigeria's debt market.

LIMITATIONS

Whilst every reasonable care and due diligence have been observed to ensure utmost professional standards in the course of preparing this investigation, its findings shall, without prejudice, be limited by the veracity of the data upon which such findings are deduced and by the extent to which verbal representations made by field experts and market practitioners are reliable.

REFERENCES

- Ang, A., and M. Piazzesi, (2001), "A no-arbitrage vector Auto regression of term structured dynamics with macroeconomic and latent variables", National Bureau of Economic Research Working Paper No. 8363
- Ameida, C.I.R., A.M. Duarte and C.A.C. Fernandes, (1998), "Decomposing and Simulating the Movements of the Term structure of Interest rates in Emerging Eurobond Markets", *Journal of Fixed Income*, 1, 21-31.
- Barrett, B., T. Gosnell and A. Heuson, (1995), "Yield curve shifts and the selection of immunization Strategies", *Journal of fixed Income*, 5 (2), 53 – 64
- Brown, R, H., S. M. Schaefer, (1994), "The Term Structure of Real Interest Rates and the Cox, Ingersoll, Ross Model", *Journal of Financial economics*, 35(1) 3-42
- Choudhry, M., (2001), *Bond Market Securities*, Prentice hall, New York
- Choudhry, M., (2004), *Analyzing and interpreting the yield curve*, Wiley and Sons, Chichester
- Elliot, D.J., (2004), *Pension Benefit Guarantee Corporation: A Yield Curve primer*, Center on Federal Financial Institutions, Washington
- Estrella. A., (2005a), "The Yield Curve and Recessions. How the difference between long-term and short-term interest rates anticipates recessions in the United States and other industrial economies", *The International Economy*, Summer
- Estrella. A., (2005b), *Why Does the Yield Curve Predict Output and Inflation?* *The Economic Journal* (July): 722 -44
- Estrella. A, and Mishkin, F., (1995), *The Term Structure of Interest Rates and its Roles in Monetary Policy for the European Central Bank*. National Bureau of Economic Research Working Paper no. 5279
- Estrella. A, and Mishkin, F., (1996), *The Yield Curve as a Predictor of U.S. Recessions*, *Current Issues in Economics and Finance*, Volume 2, Federal Reserve Bank of New York, NY.
- Estrella, A. et al., (1990), *Possible Roles of the Yield Curve in Monetary Analysis*. In *Intermediate Targets and Indicators for Monetary Policy*, Federal Reserve Bank of New York, NY.
- Evans, C., and D. Marshall, (2001), "Economic Determinants of the Nominal Treasury Yield curve." Federal Reserve Bank, Chicago Working Paper 01-16.
- Fabozzi, F. J. (2006a), *Bond Markets, Analysis and Strategies*, Prentice Hall, London.

- Fabozzi, F. J. (2006b), *Fixed Income mathematics*, 4th Edition, McGraw-Hill, London.
- Fuhrer, J. C. and G. R. Moore, (1995), "Monetary Policy Trade-offs and Correlation between Nominal Interest Rates and Real Output", *American economic review* 85 (March) pp. 219 – 239
- Ilmanen, A ., and R. Iwanowski, (1997), "Dynamics of the Shape of the Yield Curve", *Journal of Fixed Income*, 7(2), 47 – 60.
- Lardic, S., P. Priaulet, and S. Priaulet, (2003), "PCA of the Yield Curve Dynamics: Questions of Methodologies", *Journal of Bond Trading and Management*, 1 (4);
- Martellini, L., and P. Priaulet, (2000), *Fixed income securities: Dynamic Methods for Interest Rate Risk Pricing and Hedging*, John Wiley & Sons, Chichester.
- Martellini, L., P. Priaulet and S. Priaulet, (2003), *fixed income-securities: Valuation, Risk Management and Portfolio strategies*, John Wiley and Sons, Chichester.
- Mishkin. F., (1990), What Does the Term Structure Tell Us About Future Inflation? *Journal of Monetary Economics* No.25, 77 – 95
- Papageorgiou, N., and F. S. Skinner, (2002), "Predicting the Direction of Interest Rates Movements", *Journal of Fixed Income*, 11(4), 87 – 95.
- Sholarin, O.O. (2009a), *The 2007-2009 Global Financial Crisis: Causes, Consequences and policy implications*. Working paper, Pablo De-Olavide University, Seville, Spain.
- Sholarin, O.O. (2009b), *An Anatomy of Disaster: A comprehensive analysis of the 2007-2009 Global Financial-Liquidity Crises*. 7th Infinity Conference on Global Finance, Dublin, 2009
- Waggoner, F.D., (1997), *Spline Methods for Extracting Interest Rate Curves From Coupon Bond Prices*, Working Paper, Federal Reserve Bank of Atlanta.
- Watson. M., (1991), *Using Econometric Models to Predict Recessions*. Federal Reserve Bank of Chicago Economic Perspectives 15 no. 6
- Willner, R., (1996), "A New tool for Portfolio Managers: Level, Slope and Curvature Durations", *Journal of Fixed Income*, 6(1), 48 – 59.
- Wu Tao, (2003), *What Makes the Yield curve Move?* Research Department, Federal Reserve Bank of San Francisco

Websites Visited

Websites of Treasury Bond Issuers and central banks

www.treas.gov
www.francetresor.gouv.fr
www.bankofengland.co.uk
www.federalreserve.gov
www.ecb.int
www.bankofcanada.ca

Websites of data information providers

www.bloomberg.com
www.reuters.com
www.yieldcurve.com
www.bondsonline.com
www.bondmarkets.com
www.yieldcurve.com

APPENDICES

APPENDIX 1

FORWARD MATHEMATICS

Forward Rate Calculation for Nigeria's Zero-Coupon Bonds

3 - 6 Months

$$(1 + 2.52) \times [1 + F_0(3_m, 6_m)] = (1 + 3.64)^2$$

$$[1 + F_0(3_m, 6_m)] = \frac{(1+0.0364)^2}{1+0.0252}$$

$$[1 + F_0(3_m, 6_m)] = \frac{(1.0364)^2}{1.0252}$$

$$[1 + F_0(3_m, 6_m)] = \frac{1.07412496}{1.0252}$$

$$[1 + F_0(3_m, 6_m)] = 1.047722357$$

$$F_0(3_m, 6_m) = 0.047722357 = 4.77\%$$

3 Month - 1 Year

$$(1 + 2.52) \times [1 + F_0(3_m, 12_m)]^3 = (1 + 4.06)^4$$

$$[1 + F_0(3_m, 12_m)]^3 = \frac{(1.0406)^4}{1.0252}$$

$$[1 + F_0(3_m, 12_m)]^3 = \frac{1.172560571}{1.0252}$$

$$[1 + F_0(3_m, 12_m)]^3 = 1.143738364$$

$$[1 + F_0(3_m, 12_m)] = 1.045316504$$

$$F_0(3_m, 12_m) = 0.045316504 = 4.5\%$$

3 Month - 3 Years

$$(1 + 2.52) \times [1 + F_0(3_m, 3_y)]^{11} = (1 + 6.24)^{12}$$

$$[1 + F_0(3_m, 3_y)]^{11} = \frac{(1+6.24)^{12}}{1.0252}$$

$$[1 + F_0(3_m, 3_y)]^{11} = \frac{(1.0624)^{12}}{1.0252}$$

$$[1 + F_0(3_m, 3_y)]^{11} = \frac{2.067553443}{1.0252}$$

$$[1 + F_0(3_m, 3_y)]^{11} = 2.016731802$$

$$[1 + F_0(3_m, 3_y)] = 1.065848028$$

$$F_0(3_m, 3_y) = 0.065848028 = 6.58\%$$

3 Month - 5 Years

$$(1 + 2.52) \times [1 + F_0(3_m, 5_y)]^{19} = (1 + 7.10)^{20}$$

$$[1 + F_0(3_m, 5_y)]^{19} = \frac{(1+7.10)^{20}}{1.0252}$$

$$[1 + F_0(3_m, 5_y)]^{19} = \frac{(1.0710)^{20}}{1.0252}$$

$$[1 + F_0(3_m, 5_y)]19 = \frac{3.942660816}{1.0252}$$

$$[1 + F_0(3_m, 5_y)]19 = 3.845747967$$

$$[1 + F_0(3_m, 5_y)] = 1.073466422$$

$$F_0(3_m, 5_y) = 0.073466422 = 7.35\%$$

3 Month - 10 Years

$$(1 + 2.52) \times [1 + F_0(3_m, 10_y)]39 = (1 + 7.13)40$$

$$[1 + F_0(3_m, 10_y)]39 = \frac{(1+7.13)^{40}}{1+0.0252}$$

$$[1 + F_0(3_m, 10_y)]39 = \frac{(1.0713)^{40}}{1.0252}$$

$$[1 + F_0(3_m, 10_y)]39 = \frac{15.7196793}{1.0252}$$

$$[1 + F_0(3_m, 10_y)]39 = 15.3332988$$

$$[1 + F_0(3_m, 10_y)] = 1.07250892$$

$$F_0(3_m, 10_y) = 0.07250892 = 7.25\%$$

3 Month - 20 Years

$$(1 + 2.52) \times [1 + F_0(3_m, 20_y)]79 = (1 + 8.69)80$$

$$[1 + F_0(3_m, 20_y)]79 = \frac{(1+8.69)^{80}}{1.0252}$$

$$[1 + F_0(3_m, 20_y)]79 = \frac{(1.0869)^{80}}{1.0252}$$

$$[1 + F_0(3_m, 20_y)]79 = \frac{785.5378222}{1.0252}$$

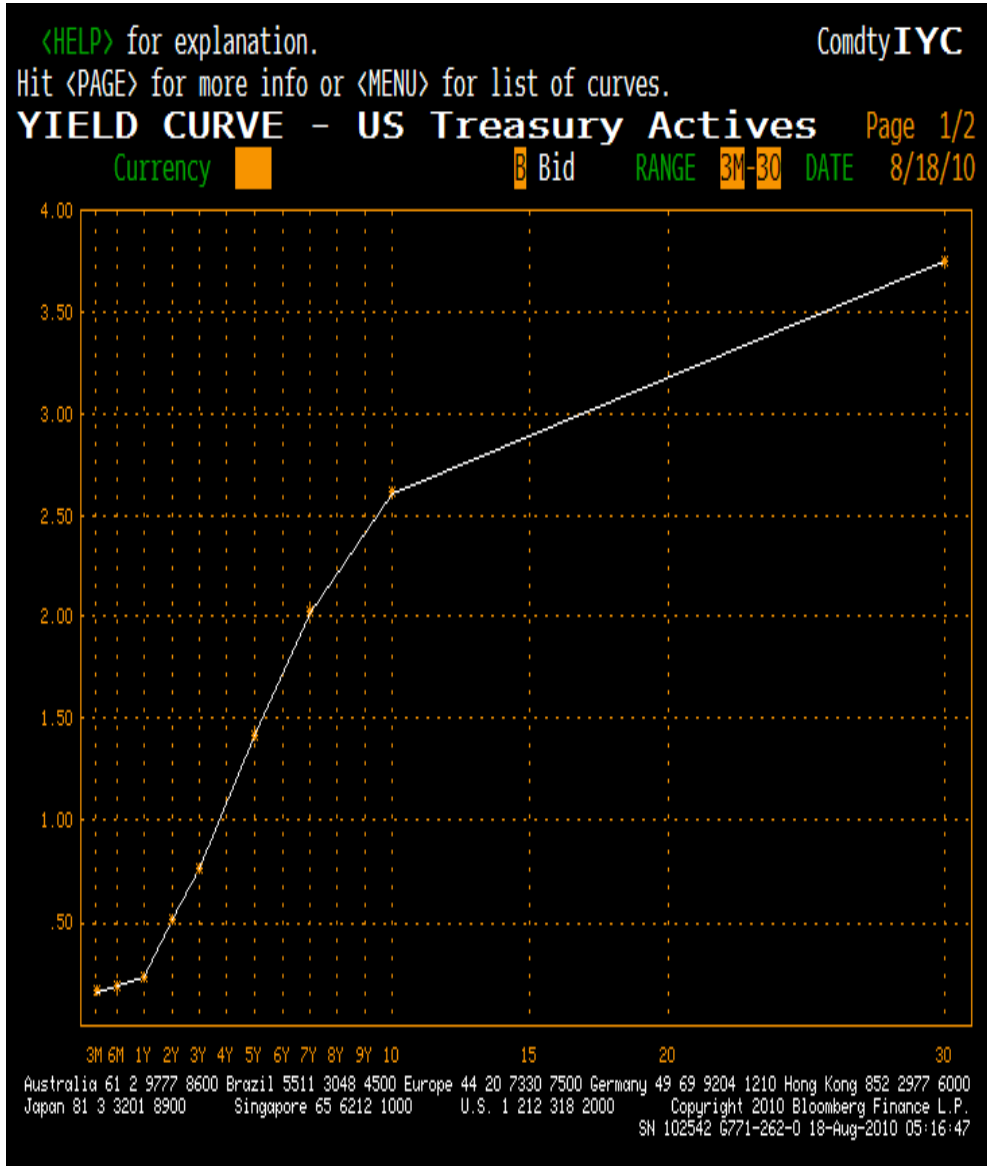
$$[1 + F_0(3_m, 20_y)]79 = 766.228855$$

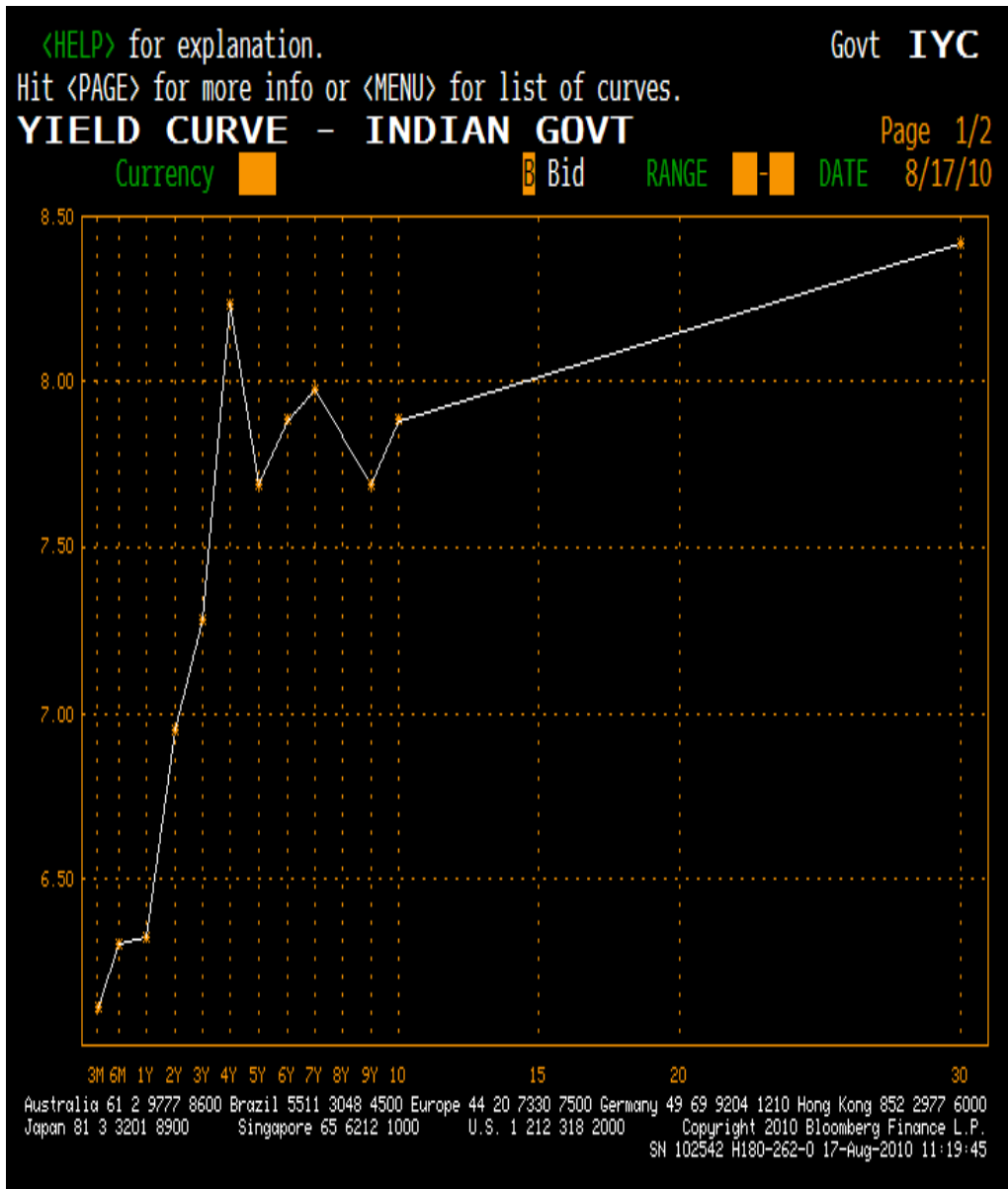
$$[1 + F_0(3_m, 20_y)] = 1.087704354$$

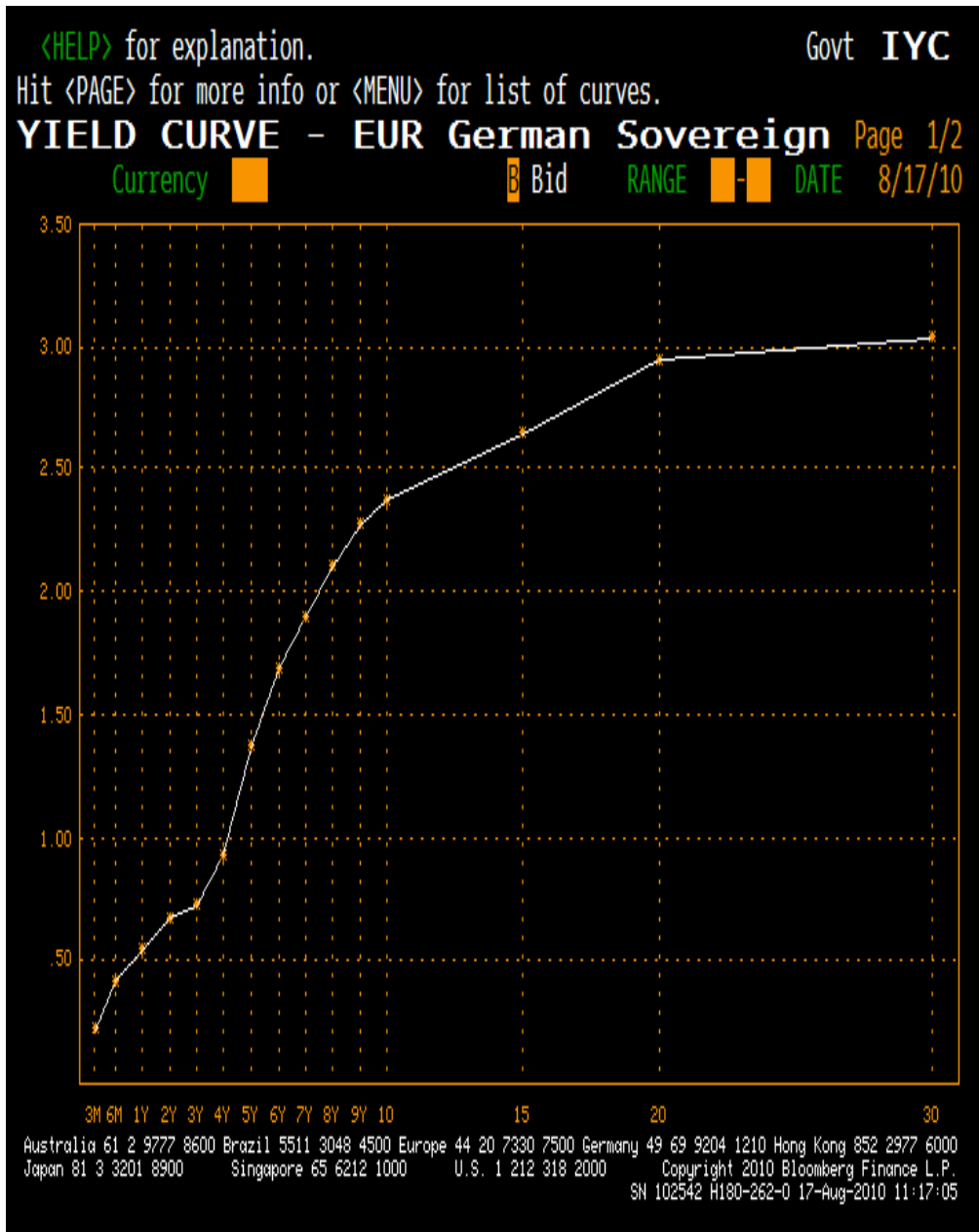
$$F_0(3_m, 20_y) = 0.087704354 = 8.77\%$$

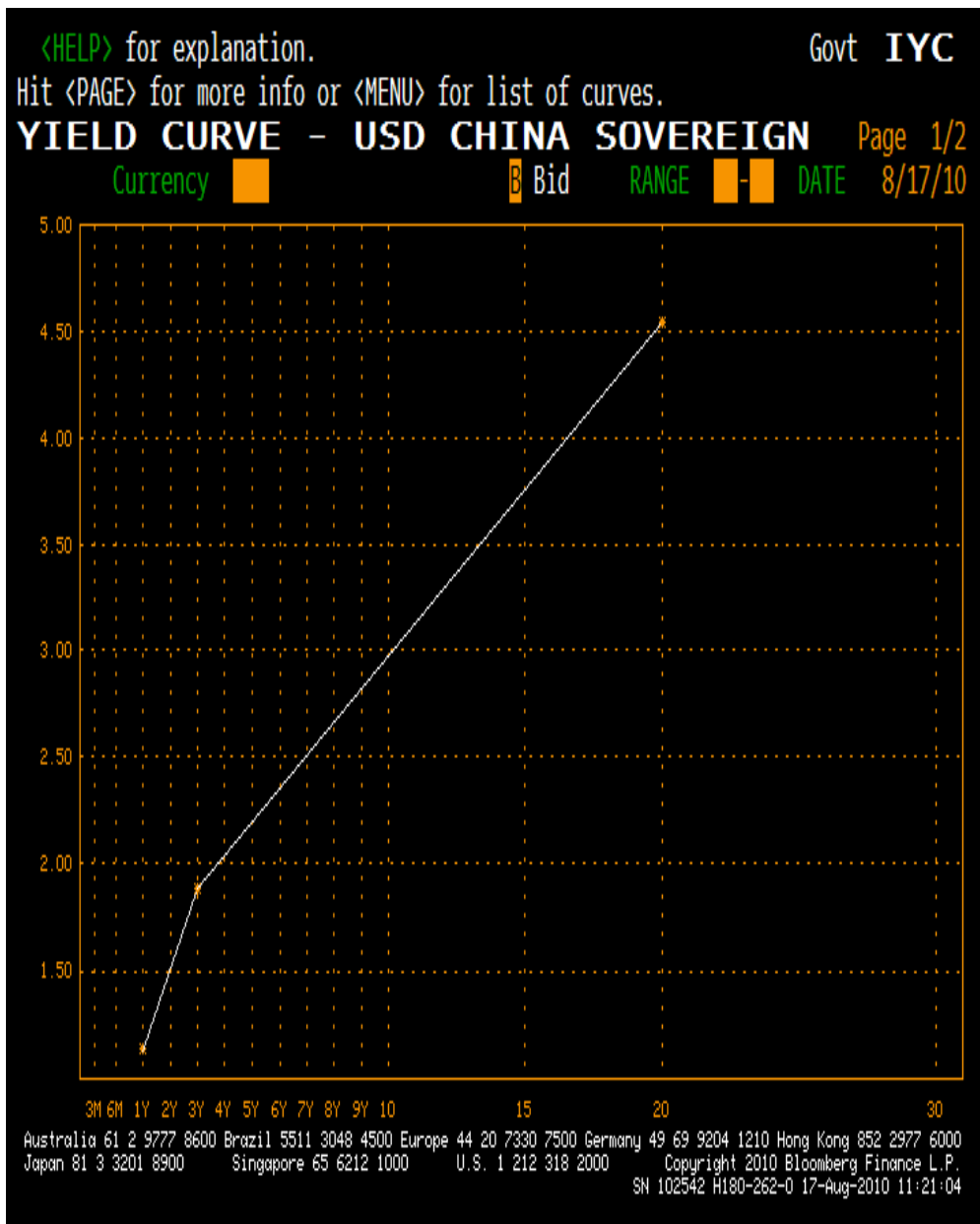
APPENDIX 2

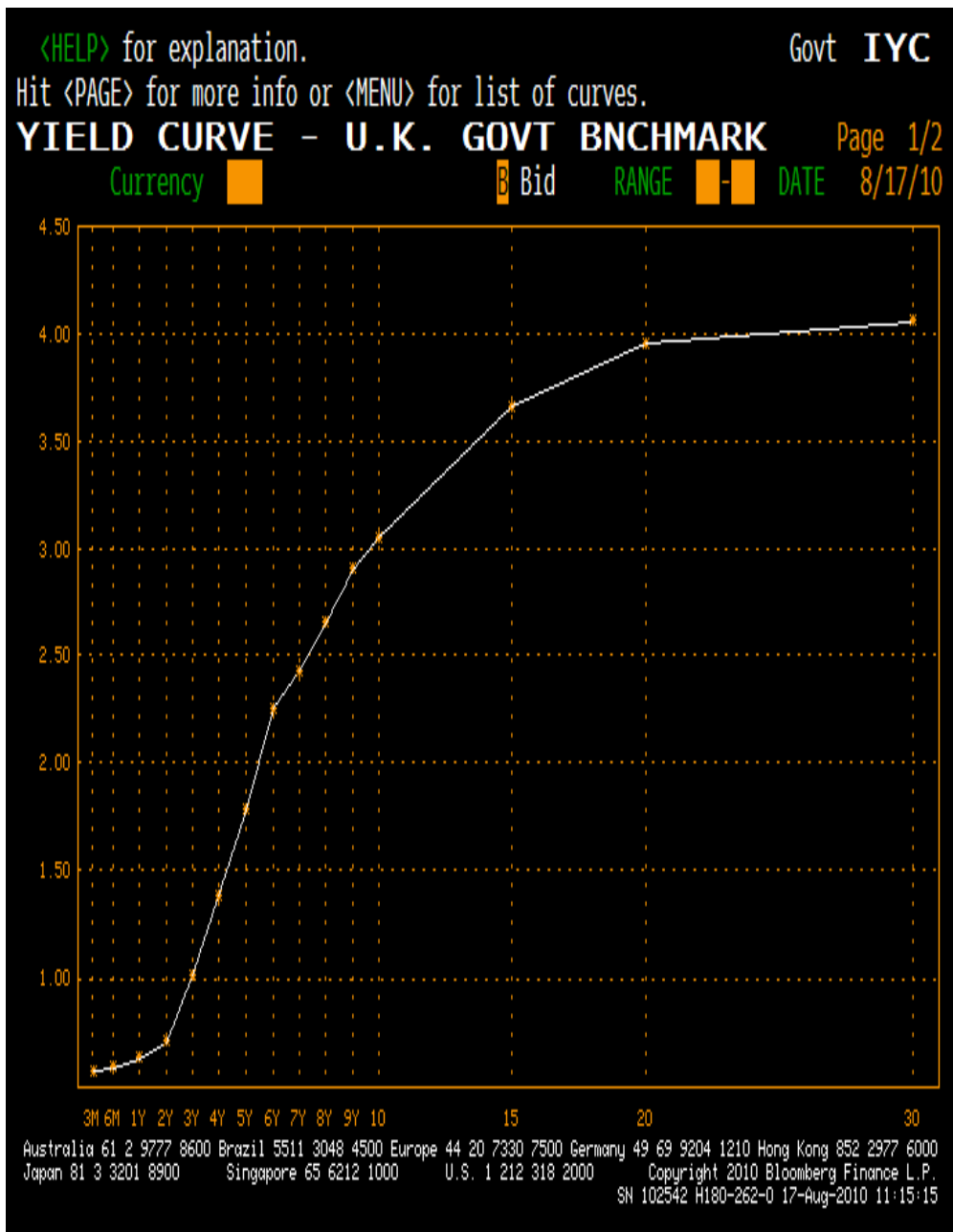
INTER-STATE SOVEREIGN YIELD CURVE COMPARISONS
[SLOPE, LEVEL AND CURVATURE ANALYSIS]

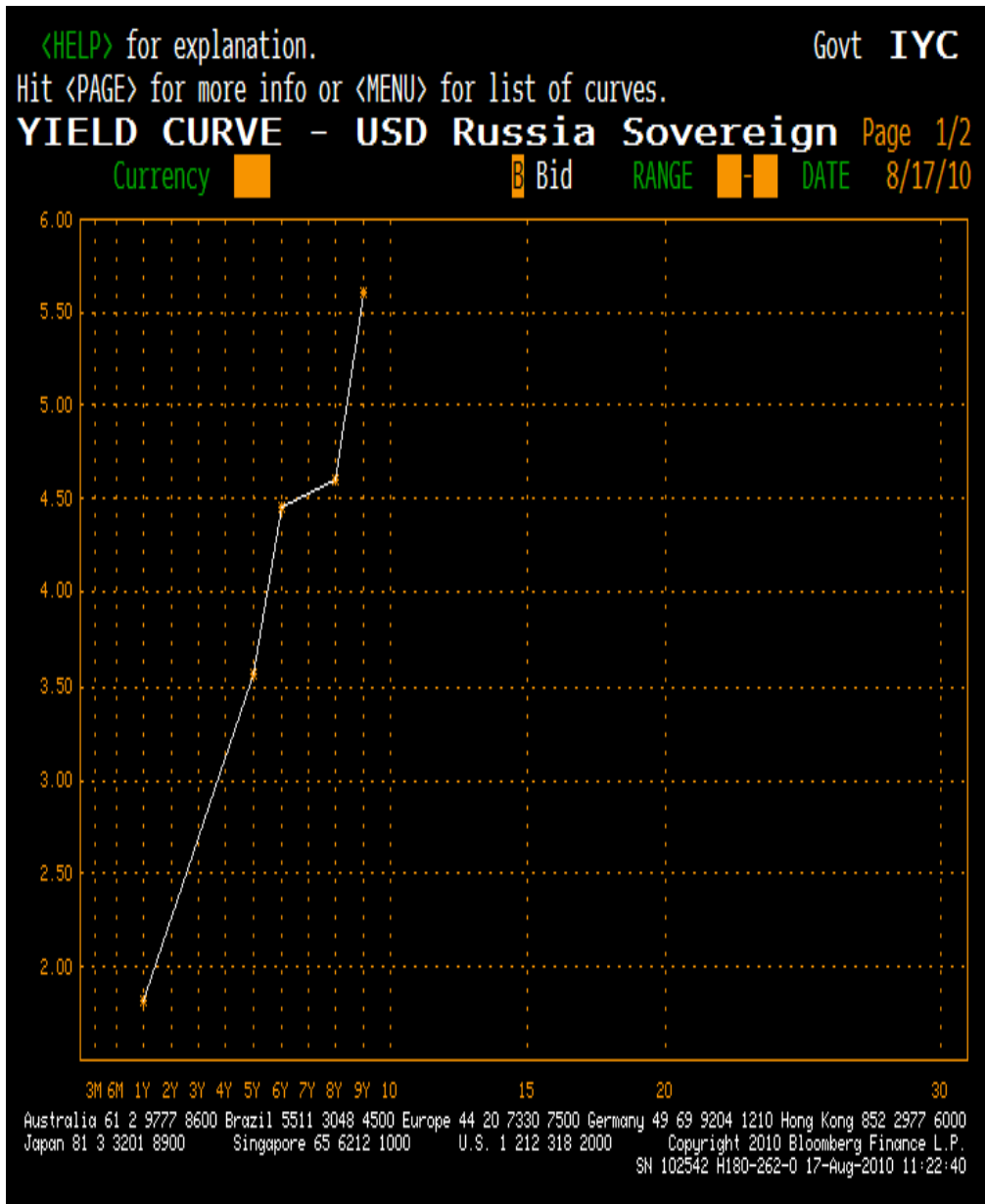


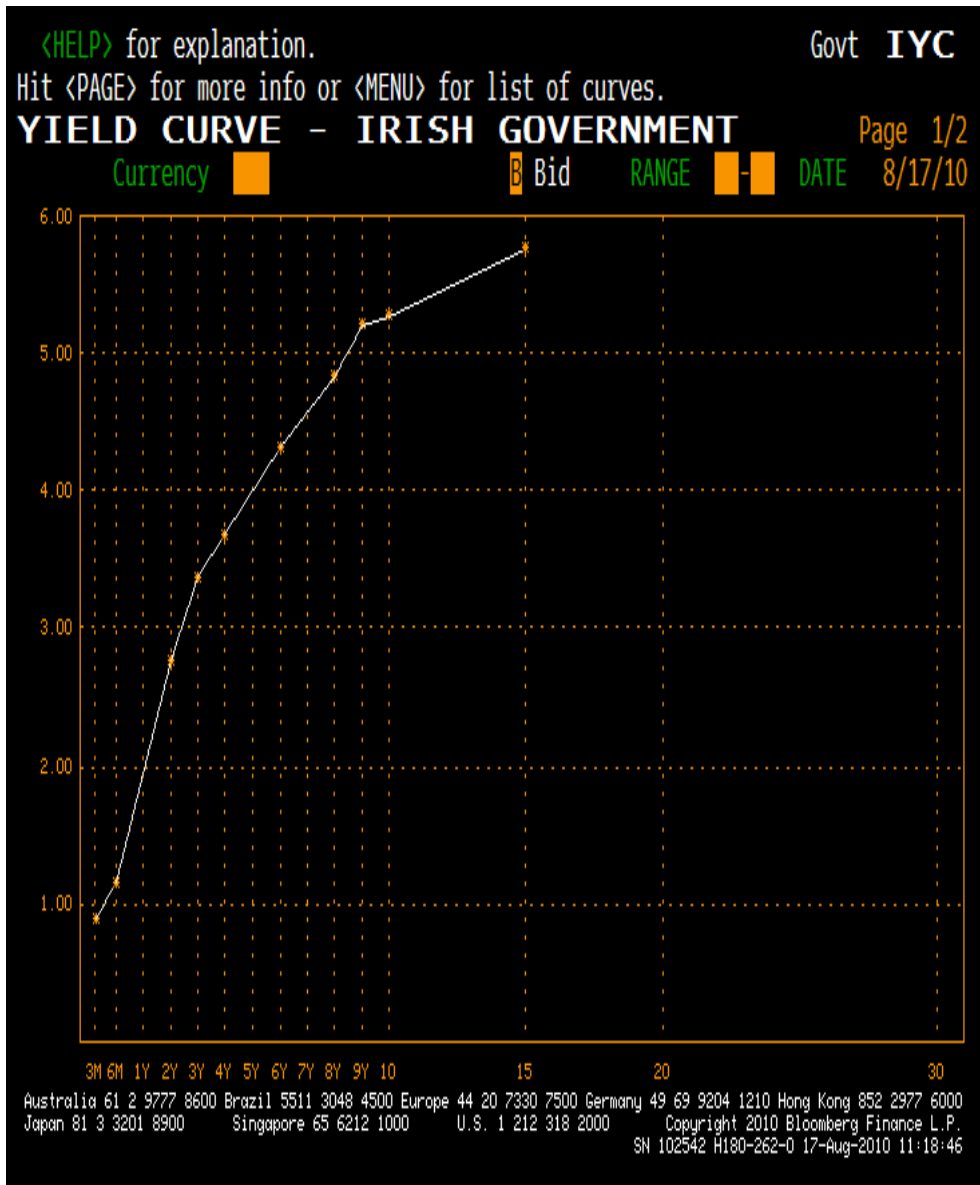


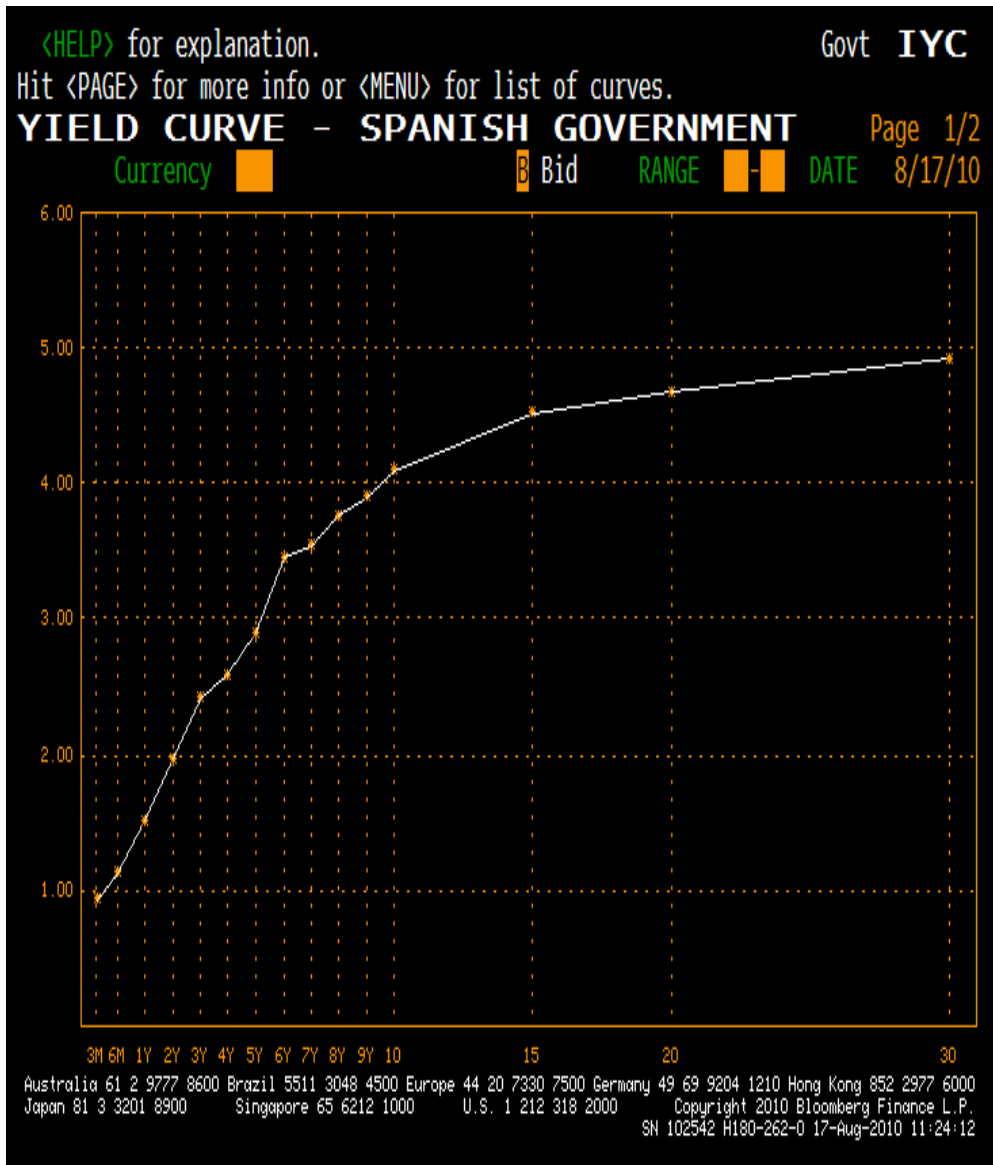


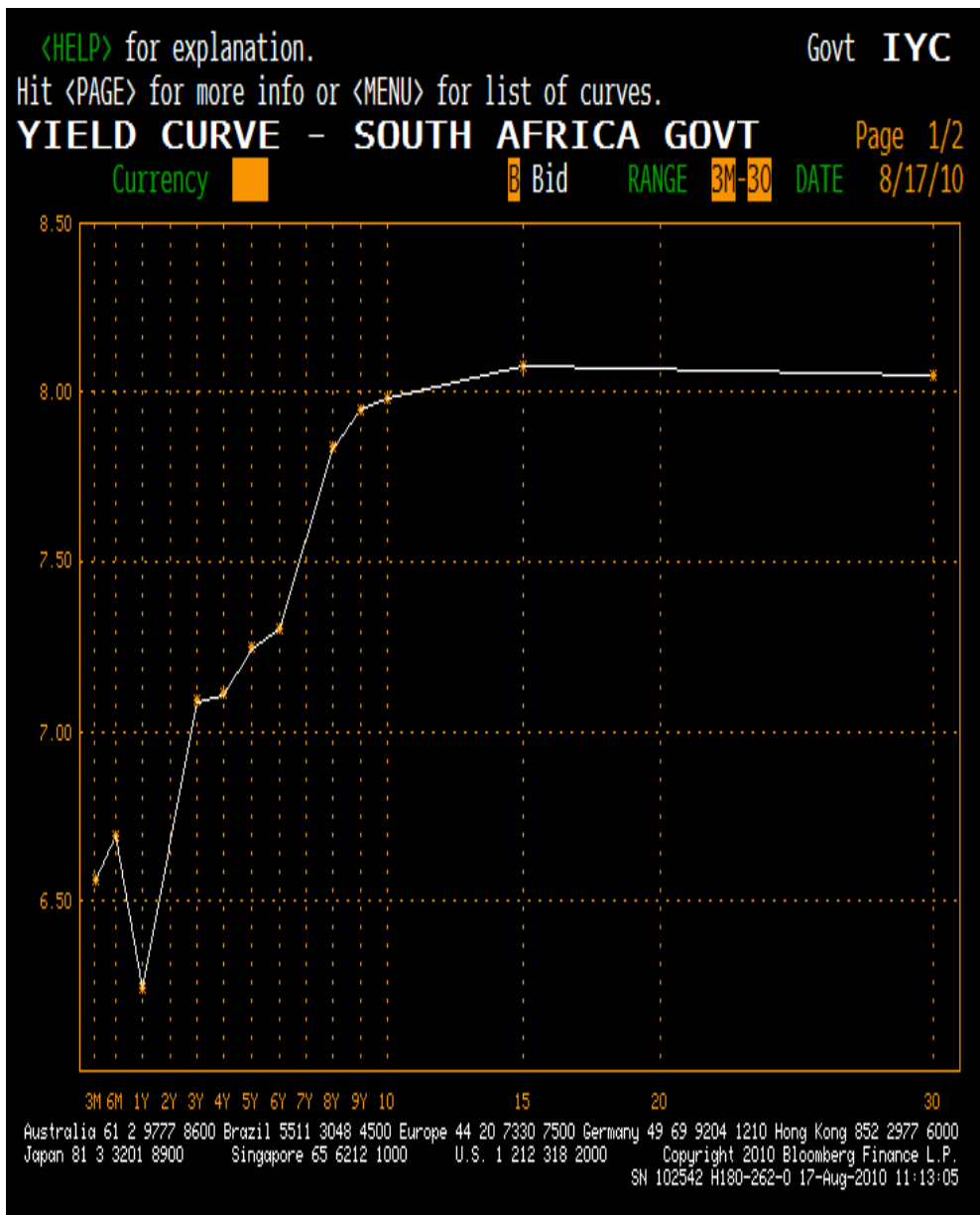


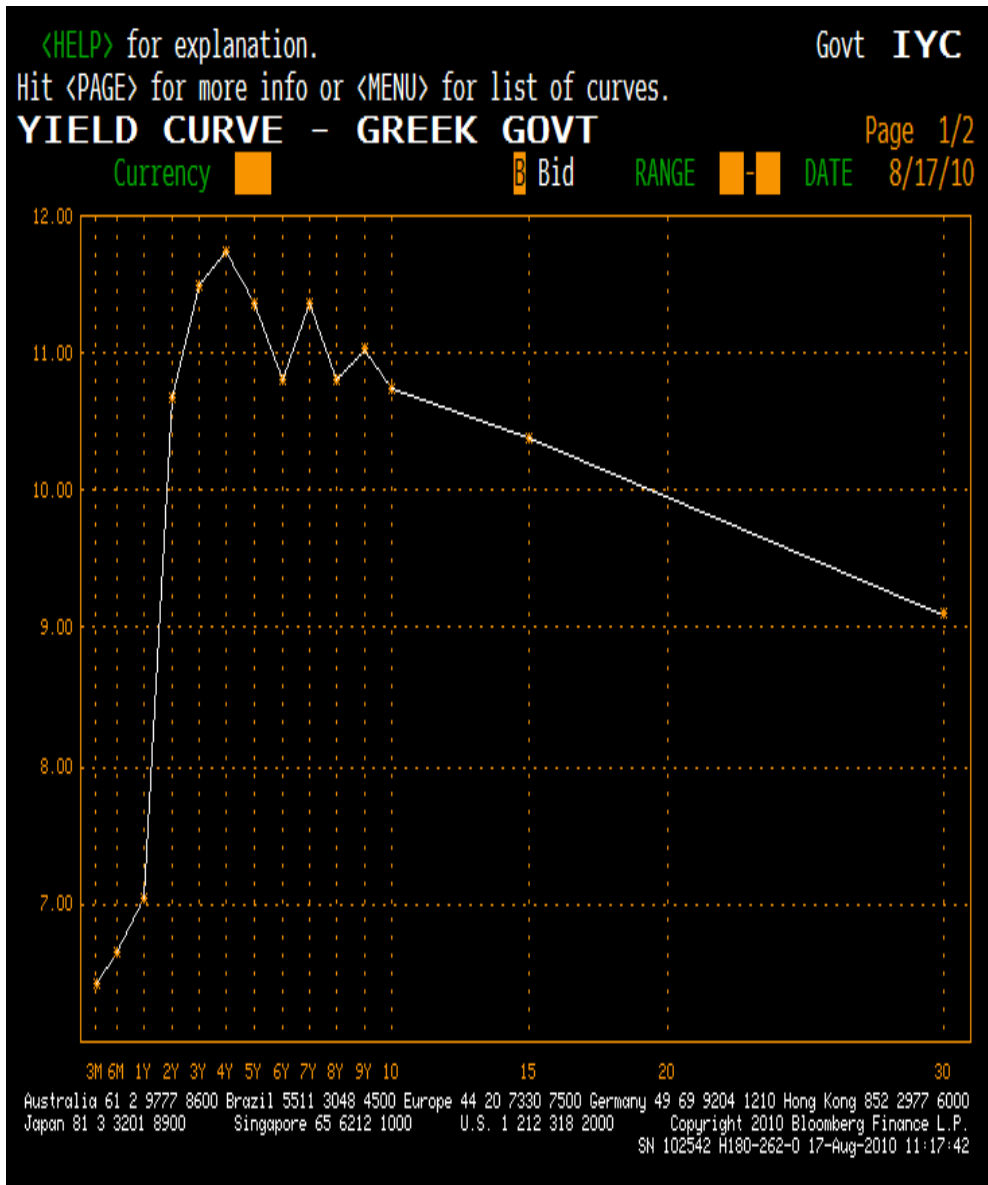


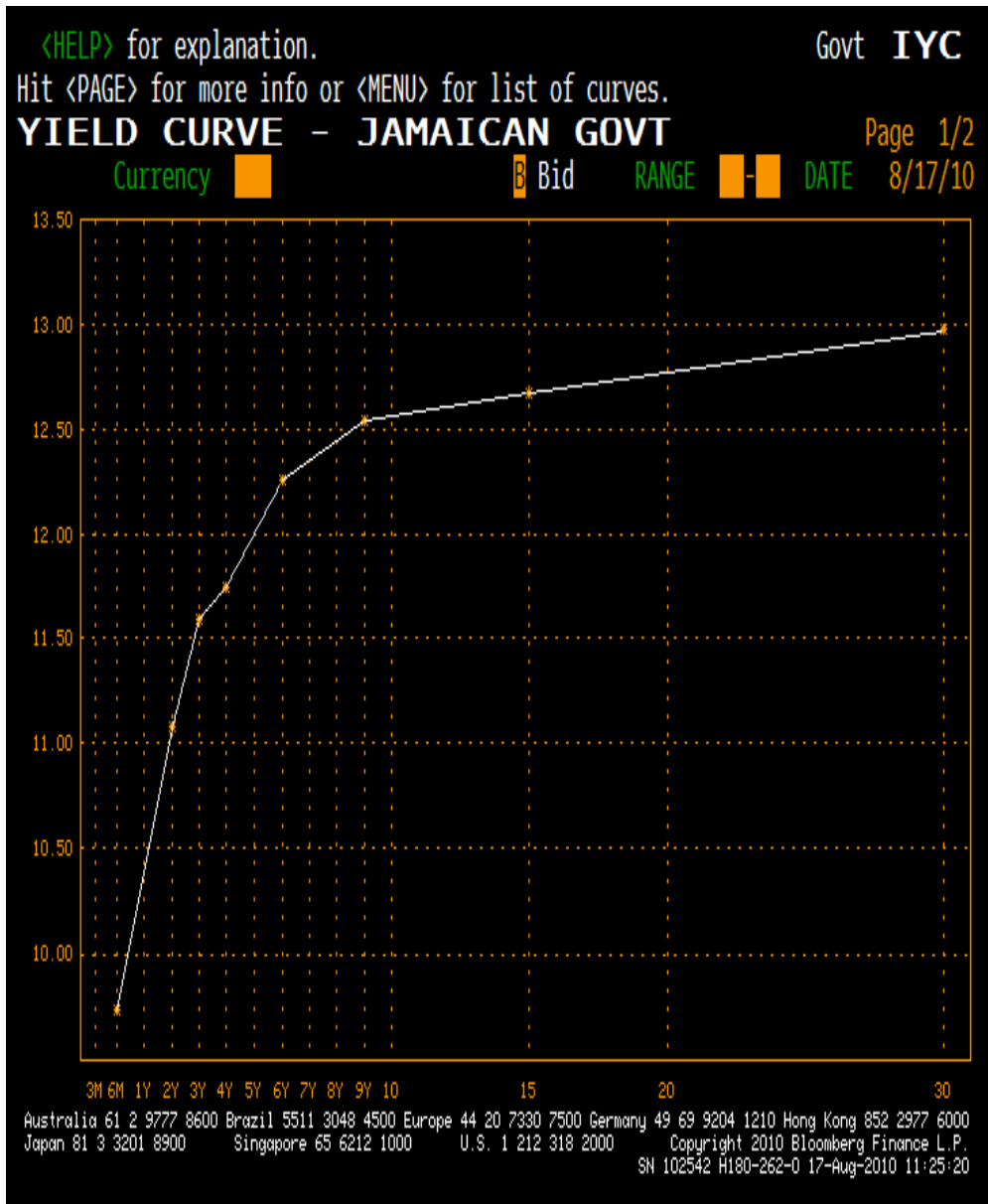




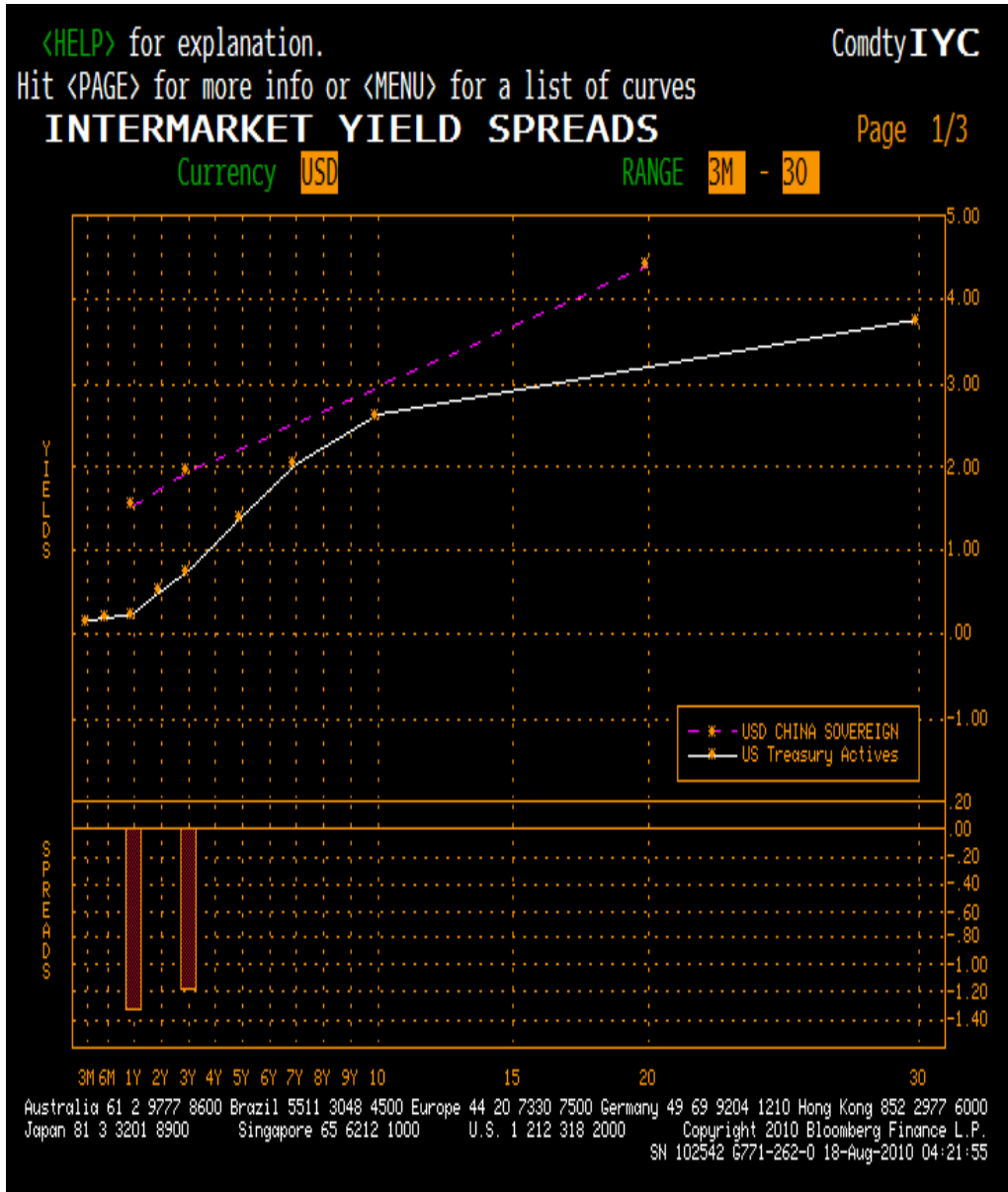




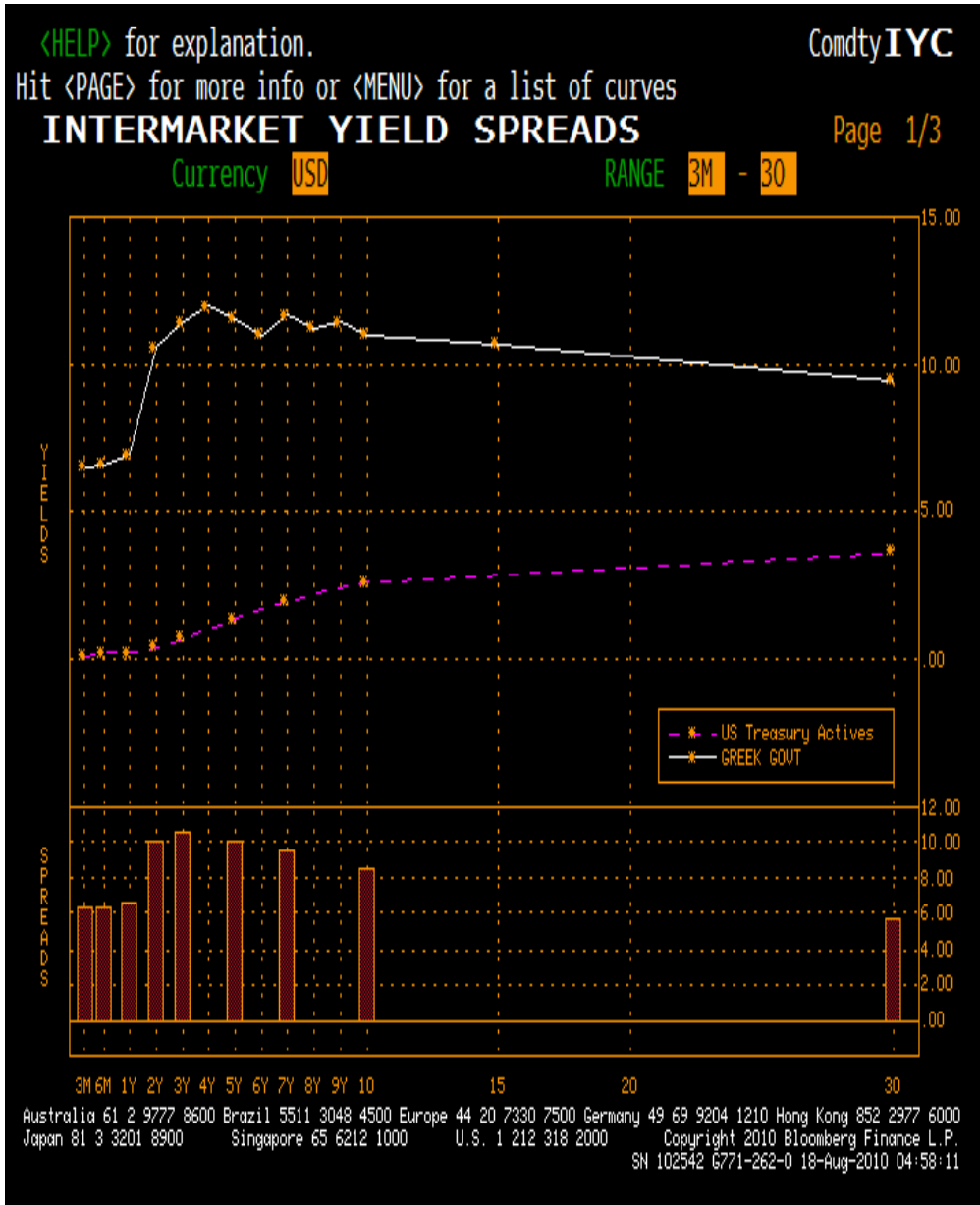




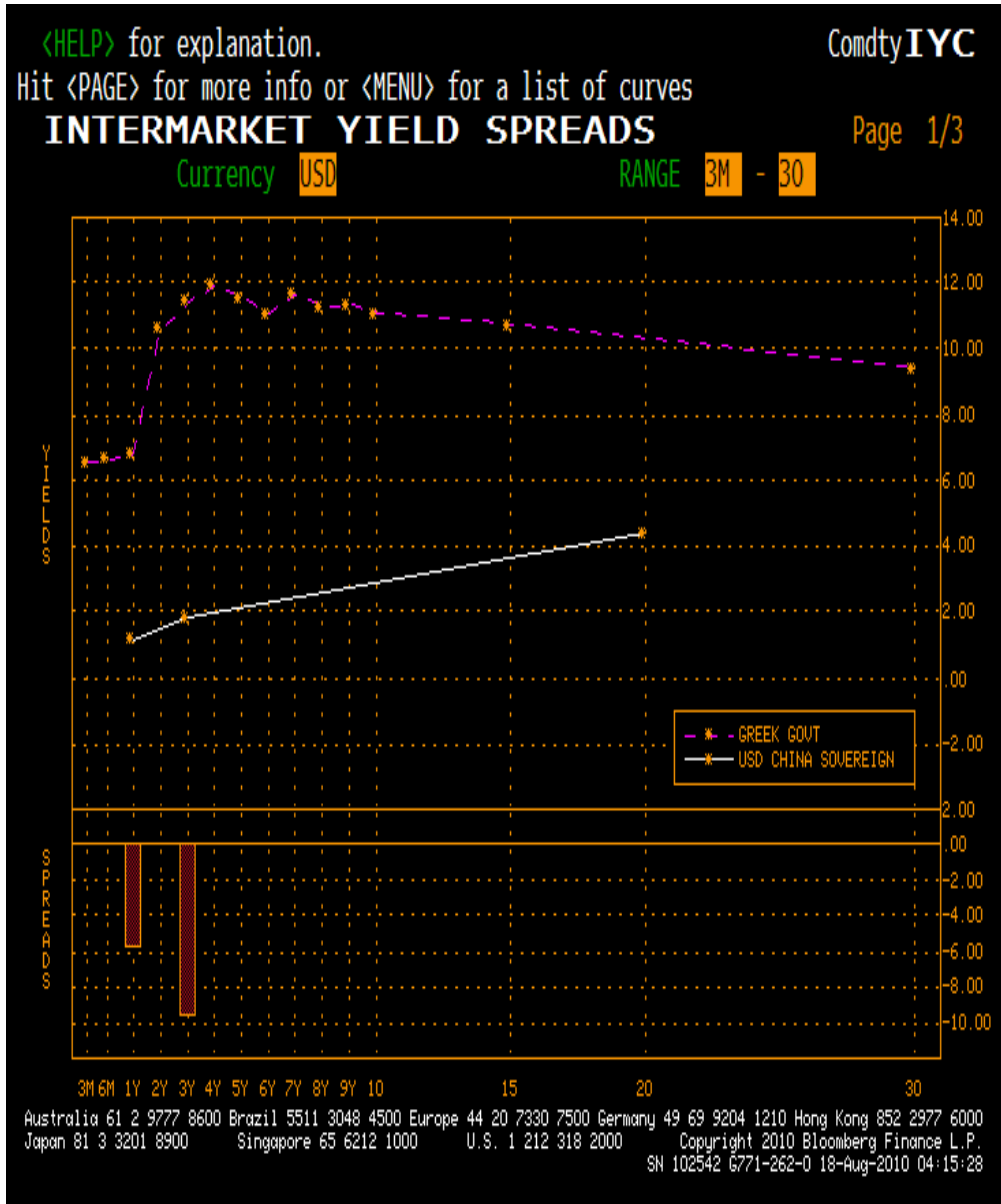
INTERMARKET YIELD SPREAD ANALYSIS
CHINA VERSUS USA



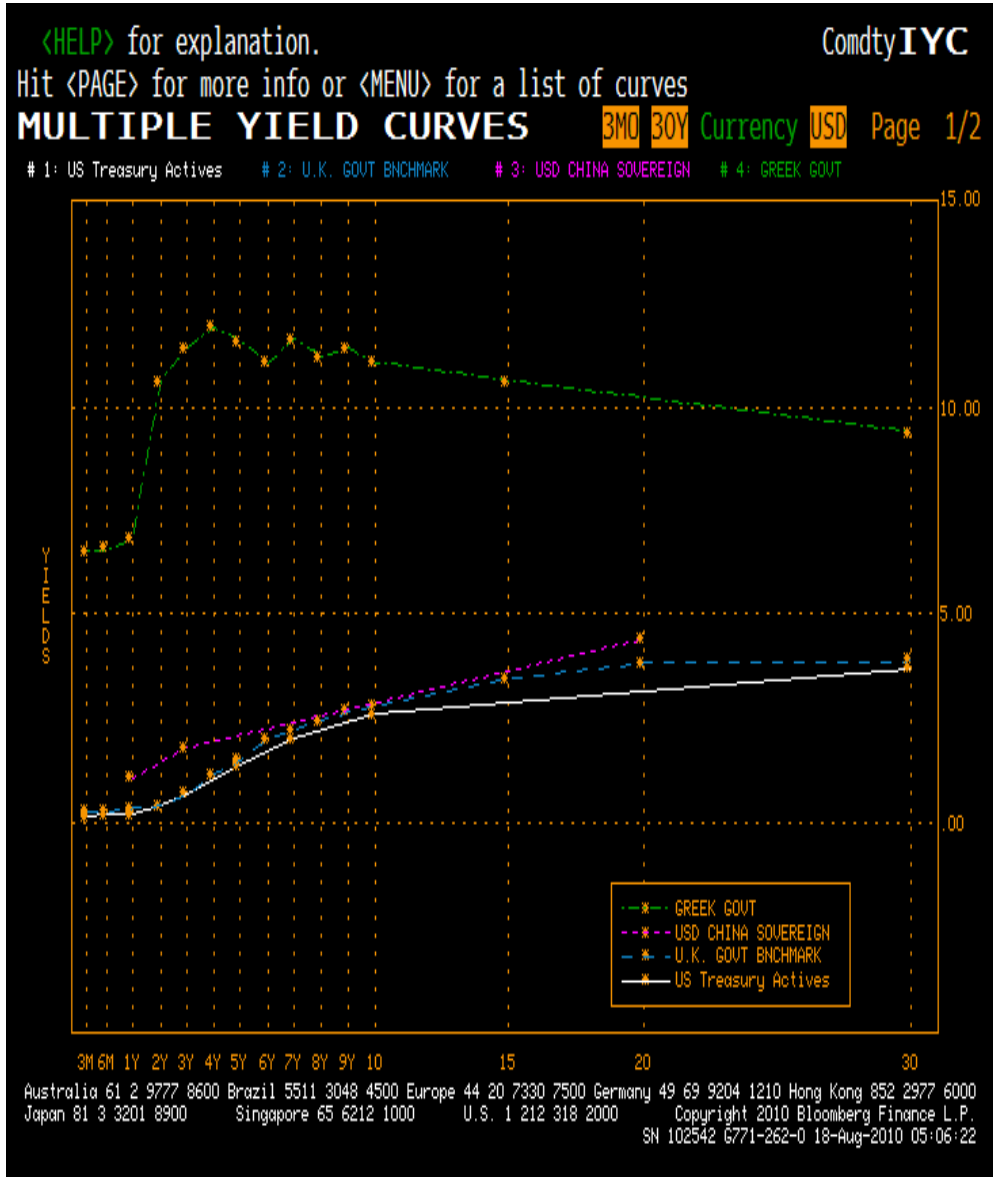
USA VERSUS GREECE



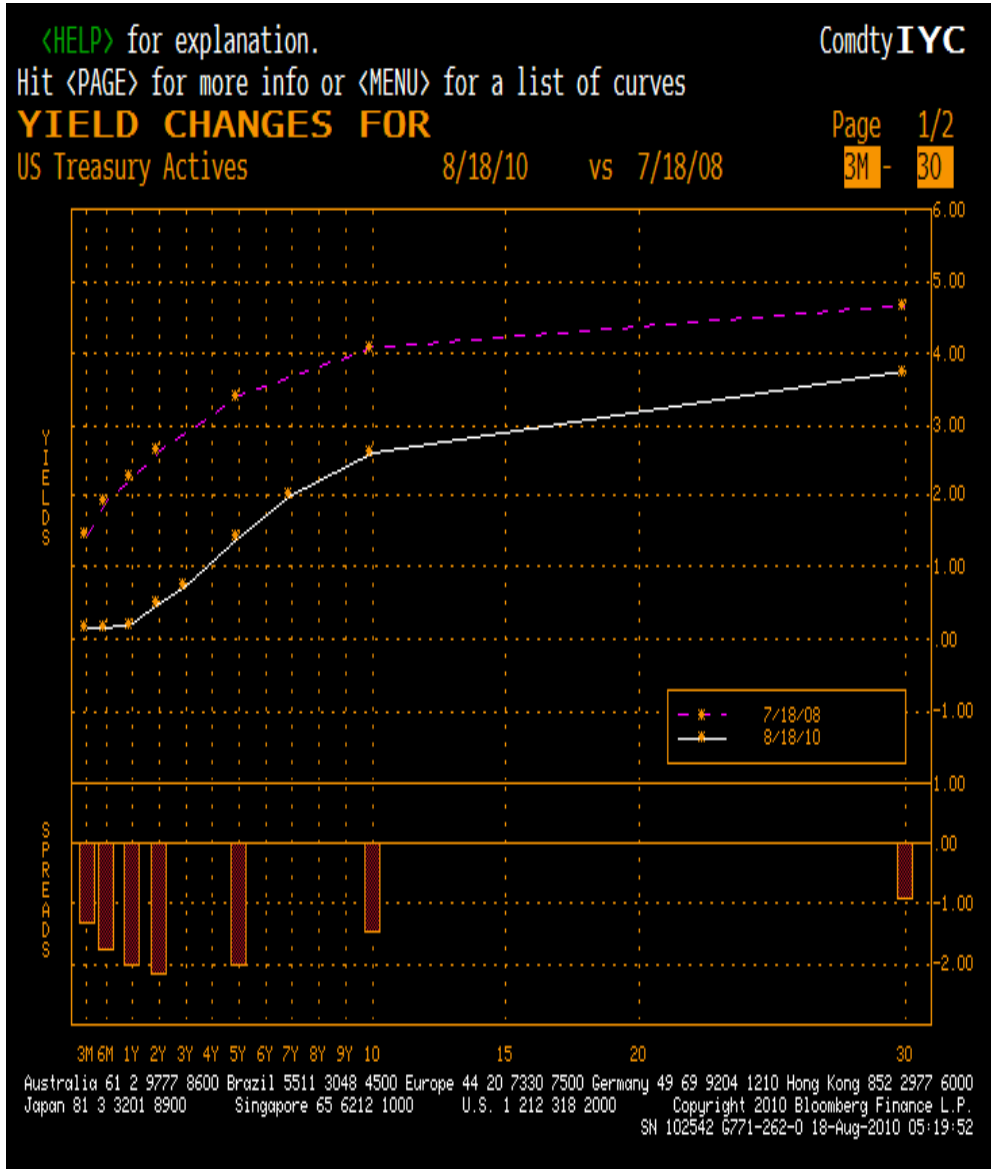
GREECE VERSUS CHINA



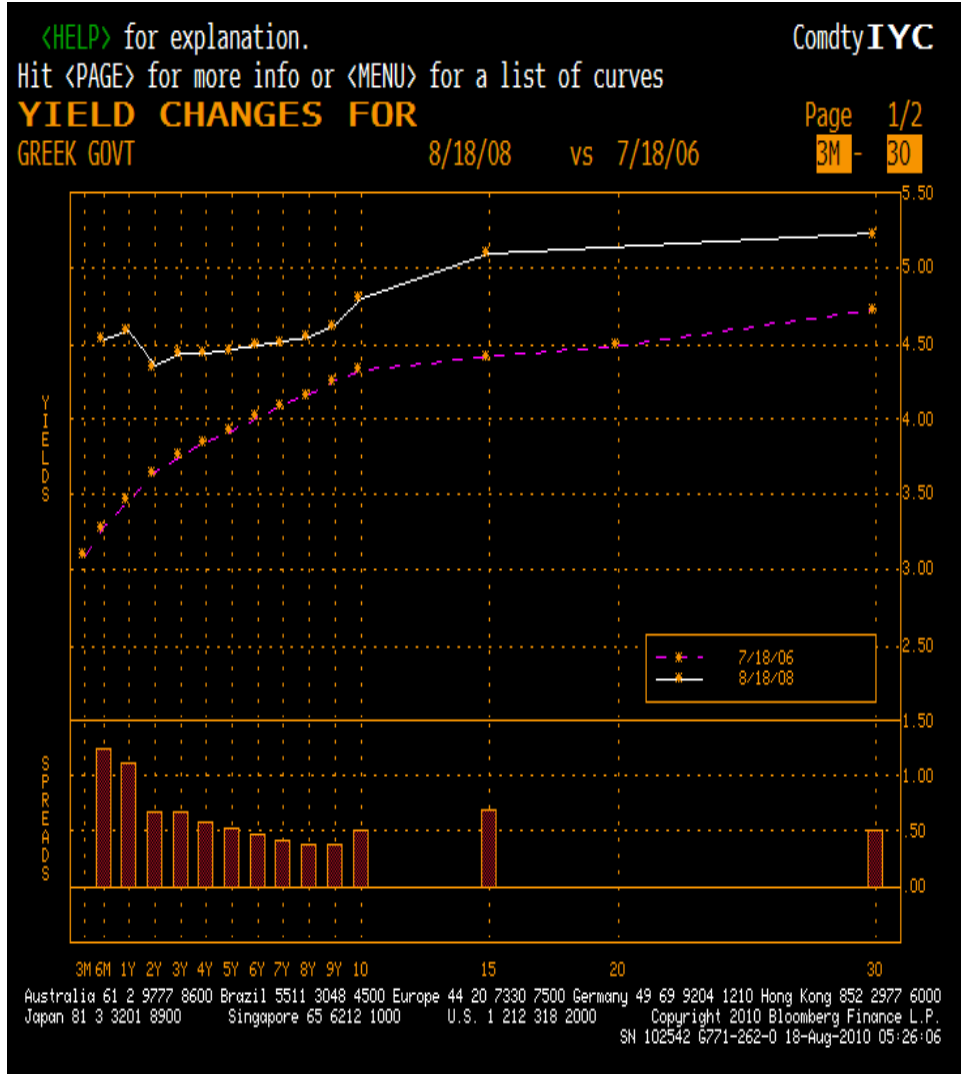
MULTIPLE YIELD CURVES ANALYSIS
[GREECE, CHINA, UK AND USA]



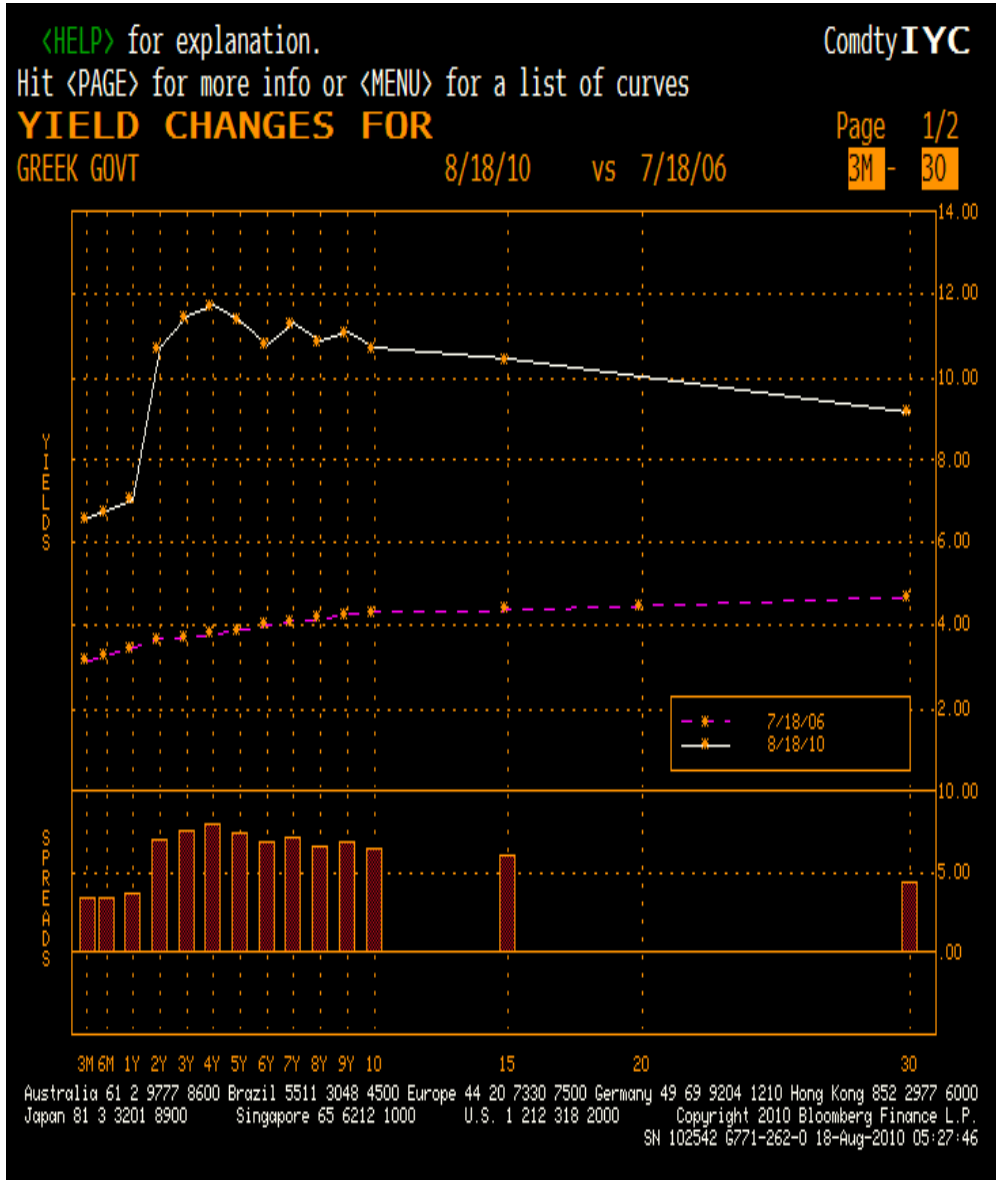
HISTORICAL SPREAD ANALYSIS FOR USA



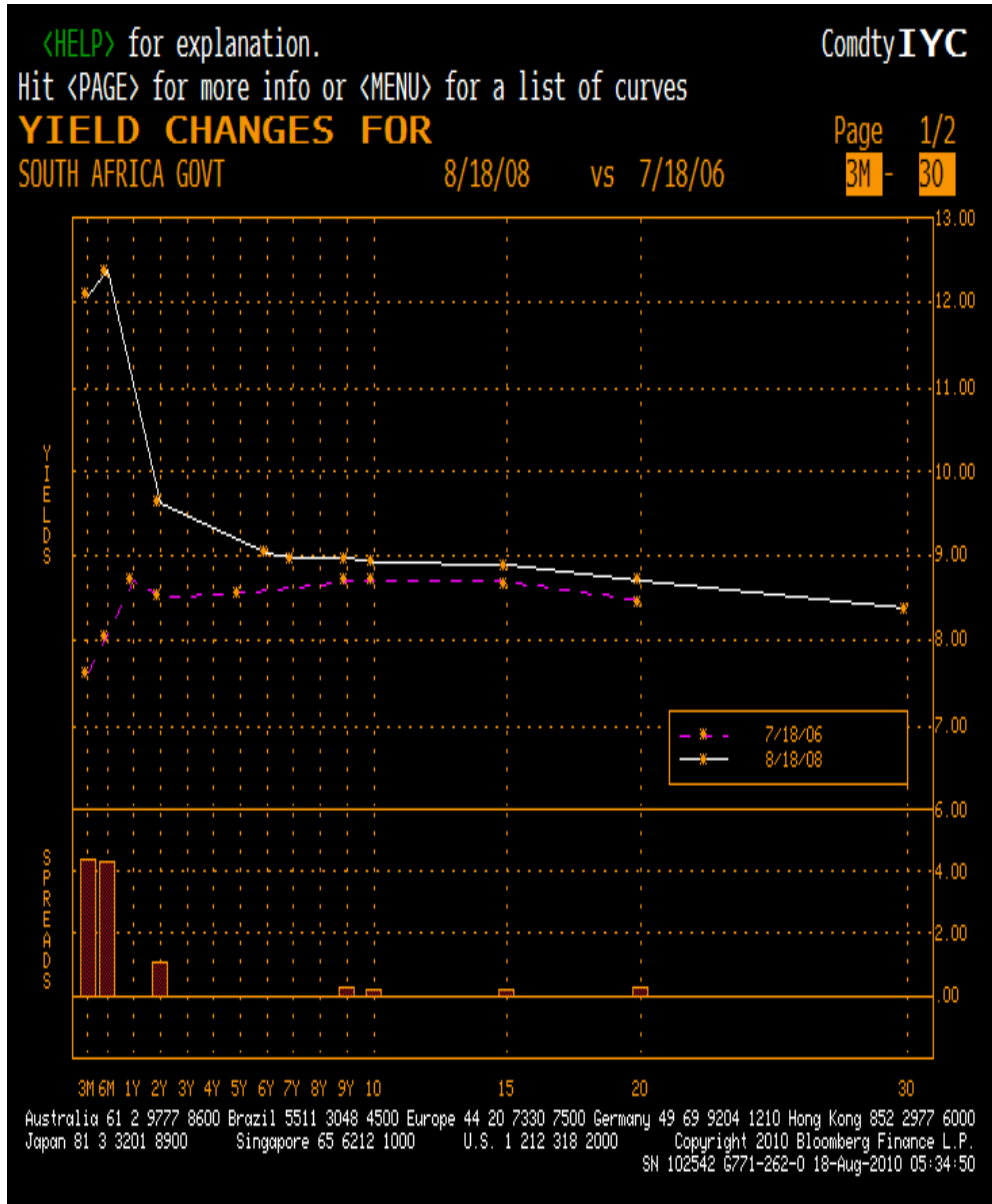
THE IMPACT OF THE SUBPRIME MORTGAGE CRISES,
AND ANATOMY OF YIELD MOVEMENTS
[GREECE BEFORE THE CRISES]



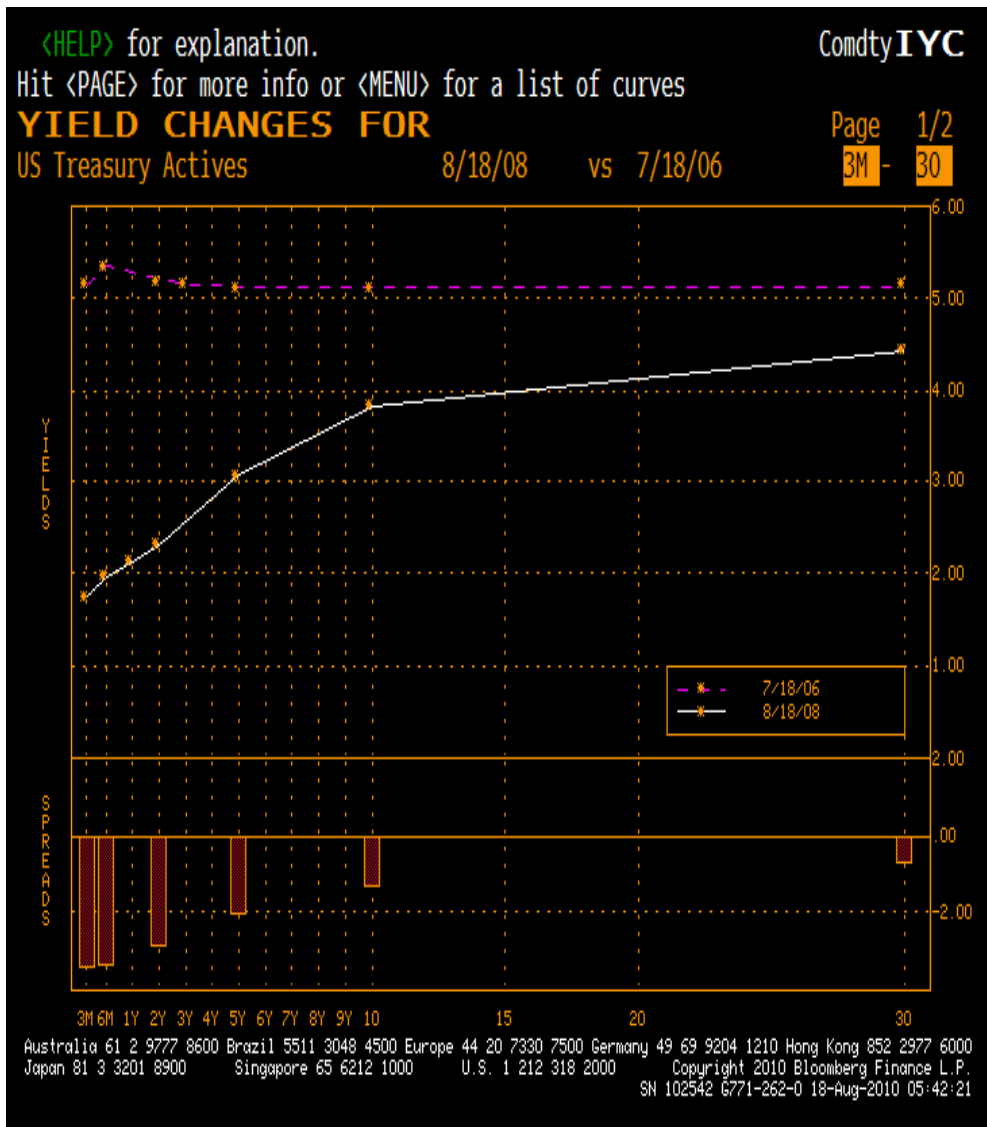
GREECE AFTER THE CRISES



SOUTH AFRICA AT THE PEAK OF THE CRISES



**YIELD CURVE DETECTS ANORMALY IN US INTEREST RATES AT THE PEAK OF THE
SUB-PRIME MORTGAGECRISES**



NIGERIA VERSUS USA

COMPARATIVE ANALYSIS OF FGN/USA YIELD VALUE AS AT JUNE 2010

Tenor	FGN ZERO-COUPON BOND YIELD	USA TREASURY BOND YIELD
91-Day	2.52	0.17
182-Day	3.64	0.21
364-Day	4.06	0.25
3-year	6.24	0.75
5-year	7.10	1.41
10-year	7.13	2.57
20-year	8.69	3.24

MULTIPLE INTERSTATE ZERO-COUPON YIELD LEVEL AND SPREAD ANALYSIS:

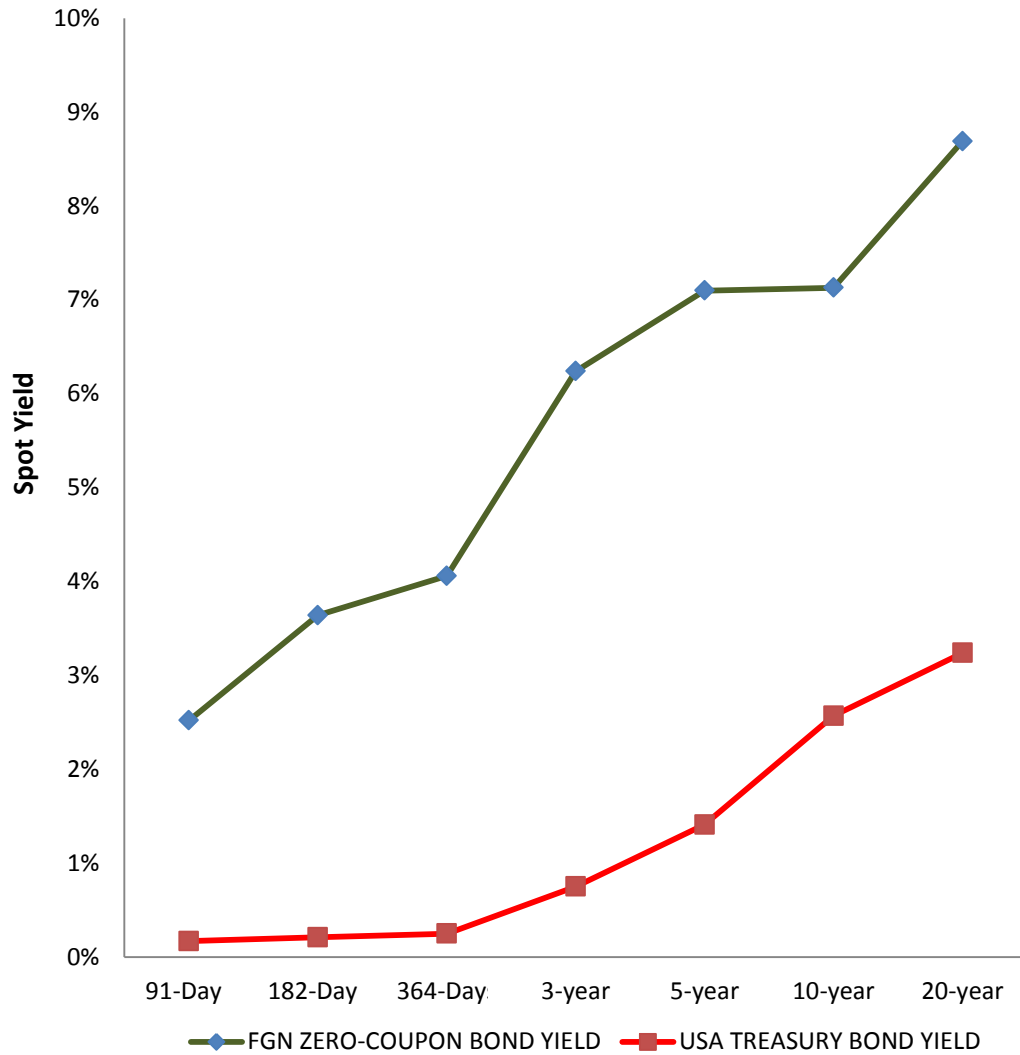
NIGERIA, REPUBLIC OF SOUTH AFRICA AND GREECE AS AT JUNE 2010

Issue date	FGN YIELD %	REPUBLIC OF SOUTH AFRICA YIELD %	GREECE %
91-Day	2.52	6.60	6.40
182-Day	3.64	6.70	6.68
364-Day	4.06	6.22	7.00
3-year	6.24	7.15	11.50
5-year	7.10	7.25	11.40
10-year	7.13	8.00	10.80
20-year	8.69	8.10	10.00

NIGERIA VERSUS THE USA

COMPARATIVE ANALYSIS OF SLOPE, LEVEL CURVATURE AND SPREAD

FGN ZERO-COUPON VS US TREASURY BOND SPREAD
AS AT JUNE 2010



APPENDIX 3

IDEAL FORMAT FOR RECORDING THE FGN BOND YIELD DATA

TENOR	0-coupon Yield % As at day n	Changes today n- (n-1)	Changes 1 wk ago n-(n-7)	Changes 1-mth ago n-(n-30)	Forward yield as at date n	Issue date
91-day						
182-day						
364-day						
3-year						
5-year						
7-year						
10-year						
20-year						
30-year						

